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Algorithms to investigate the effect of melting on the stability of icebergs.

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ALGORITHMS TO INVESTIGATE THE EFFECT OF MELTING ON THE STABILITY OF ICEBERGS

by

MALAIAPPAN VISWANATHAN

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To My Parents

ABSTRACT

An algorithm has been developed to study the effect of melting on the stability of a two dimensional irregularly shaped iceberg floating on water. The method used in the algorithm can be employed for regular symmetrical shapes such as rectangular shapes as well as for highly irregular two dimensional shapes. The algorithm is in the form of a computer program which needs as inputs the shape of the iceberg defined as co-ordinate points with respect to an arbitrarily chosen Cartesian co-ordinate system and an initial orientation of the iceberg with respect to the water surface.

Three phases of work are involved. The first part can be used for finding the various orientations at which a two dimensional irregularly shaped iceberg can float in stable equilibrium on water. It may be recalled that at stable equilibrium orientations, if the iceberg is subjected to a small angular disturbance, then a couple should be set up in such a way that it tends to restore the iceberg to the same orientation. A criterion is developed for describing the relative stability of the various orientations which can be taken by an iceberg floating at equilibrium positions.

The second phase describes a method for modelling the two degrees of freedom of motion of the iceberg. This is done by solving numerically the differential equations representing the

translational as well as the rotational motion of the iceberg. The validity of this method is established by predicting the motion of the iceberg from an initial orientation for the case of no melting. It is found that this algorithm brings the iceberg to one of the several stable equilibrium positions predicted by the first algorithm.

The third part incorporates a method for dynamically modelling the effect of melting of the iceberg. It is based on the assumption that the entire iceberg is at the melting temperature and that the coefficient of heat transfer around the surface of the iceberg is uniform. The coefficient of heat transfer is assumed to be equal to that in the case of a turbulent natural convection boundary layer. Although the melting model is relatively crude and could be improved, its incorporation in the dynamic model demonstrates how melting can affect iceberg behaviour such as roll over.

In all, three shapes are analysed, a typical irregular shape, an irregular shape with a hole in it and a rectangular shape.

PREFACE

The author would like to express his sincere gratitude to his supervisor Dr. Norman W. Wilson for his help and suggestions throughout this project. His readiness to sit down to analyse the problems and his patience in spending long hours in discussion can never be forgotten. The author acknowledges that it was a great pleasure in working with him and that he learned a lot by his excellent supervision. The discussions with Dr. Zygmunt F. Reif, for modelling the motion of icebergs were very useful and gratefully acknowledged. The financial support which made this project possible was provided by an NSERC grant No.A1243 and is gratefully remembered.

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NOMENCLATURE

A : the perpendicular distance between the lines of action of buoyancy and weight in m (moment arm)

A_o : value of A during previous iteration in m

B : the force of buoyancy in N

B_o : value of B during previous iteration in N

C_a : angular damping coefficient in N-m-sec/rad

C_d : linear damping coefficient in N-sec/m

C_1 : linear damping ratio (dimensionless)

C_2 : angular damping ratio (dimensionless)

C_p : specific heat of ice in W-sec/Kg·K

F : spring force in N

F_D : drag force in N

F_{DA} : drag moment in N-m

f_D : drag coefficient (dimensionless)

g : acceleration due to gravity in m/sec^2

h : heat transfer coefficient in W/m^2·K

I : mass moment of inertia in Kg-m^2

K : spring constant in the case of a mass, spring and damper system(linear) in N/m

K_θ : spring constant in the case of a mass, spring and damper system (angular) in N-m/rad

k : thermal conductivity of ice in W/m·K

L : characteristic dimension in m

M : mass of iceberg in Kg

P_w : frontal area in m^2

Pr : Prandtl Number (dimensionless)

Ra : Rayleigh Number (dimensionless)

T : angular displacement of iceberg about its centre of gravity in rad

T_o : value of T during previous iteration in rad

T_N : value of T obtained from linear interpolation in rad

T_{INI} : value of T at time t=0.0 in rad

t : time in sec

T_b : bulk temperature of water in degree centigrade.

T_i : temperature of ice in degree centigrade.

V_w : velocity of iceberg in water in m/sec

V : rate of melting in m/sec

W : weight of iceberg in N

WL : waterline

x,y : cartesian co-ordinates

X : function of transformation

X1,X2 : consecutive amplitudes

Y : perpendicular distance of the centre of gravity of iceberg from waterline in m

Y_o : value of Y during previous iteration in m

Y_N : value of Y obtained from linear interpolation in m

Y_{INI} : value of Y at time t=0.0 in m

Z : function of transformation

dz : melt displacement in m

α : thermal diffusivity of water in m^2/sec

β : thermal expansion coefficient in 1/K

δ : logarithmic decrement (dimensionless)

ΔT : a small increment in T in rad

ΔY : a small increment in Y in m

Δt : a small increment in t in sec

λ : latent heat of fusion of ice in J/kg

ν : kinematic viscosity in m^2/sec

ρ_w : density of water in kg/m^3

ρ : density of ice in kg/m^3

ω : angular velocity of iceberg in water in rad/sec

Subscripts :

n : time level

Symbols :

$+$: indicates the location of the centre of gravity

: indicates the location of the centre of buoyancy

1. INTRODUCTION

The rapid increase in oil and gas exploration and the probability of future development in the off-shore regions of Newfoundland and Labrador has stimulated an interest in a thorough understanding and prediction of the thermal and mechanical behaviour of icebergs. The reason for this is the hindrance to these operations caused by the presence of huge icebergs in these regions. The knowledge accumulated about the ocean, the icebergs and the atmosphere in these regions has enabled scientists to gain a more precise picture of icebergs as ensembles of three dimensional irregularly shaped masses with masses upto 2×10^6 Tons approximately. These icebergs are driven by air and water currents, influenced by the earth's rotation and subject to melting and freezing as dictated by the regional climate. This melting or freezing affects the floating stability of icebergs and it has been observed that icebergs roll over from time to time. This roll-over phenomenon causes some difficulty in the placement of human observers or equipment on or near icebergs. Therefore, a thorough understanding is necessary of the response of an iceberg when subjected to varying thermomechanical inputs.

The process of melting has a very serious impact on the above application, for melting affects the floating stability and causes roll over. In the following the process of melting as applied to an irregularly shaped solid floating in a stationary liquid medium is studied in detail.

If an isothermal solid of irregular shape is to float in a stationary liquid medium with higher density, it will do so in one of the several possible stable equilibrium positions, with part of the solid in the liquid medium and part of the solid exposed to air. If the liquid medium has a temperature different from that of the solid, then due to natural convection, heat transfer will take place from the liquid to the solid if the temperature of the liquid is greater than that of the solid, or from the solid to the liquid medium if the temperature of the solid is greater than that of the liquid. Similar heat-transfer takes place between the air and the solid. If the transfer of heat to the solid is such as to melt the solid, then the shape of the solid changes as time progresses. The change in shape of the solid affects its floating stability to the extent that it will be continually moving to a small extent in the liquid medium. In some cases a previously stable shape gets modified by melting to the extent that stability is not obtainable by a small motion and the object rolls over to a very different orientation. While icebergs are in fact three dimensional irregularly shaped objects an analysis will become simpler if an isothermal iceberg of infinite length and of highly irregular cross-section is considered. This is referred to in this report as a two-dimensional shape.

Generally in regions where icebergs occur, the air temperatures are low and the amount of heat gained by the ice from the surrounding water is much greater than that gained from the air. Therefore, the change in shape takes place at a much faster rate in the portion of the iceberg which is below the water surface. This change in shape may render the present floating position unstable and the iceberg may eventually roll over to a new stable equilibrium position.

Broadly speaking, the present work is concerned ~~with~~ modelling the dynamic stability of a two dimensional iceberg. However, before defining the objective of the present work in a greater detail, a survey of the relevant literature is necessary.

As far as is known, the first work on the melting of an ice block was done by Tkachev (1). He performed experiments by melting ice spheres and cylinders in both horizontal and vertical positions with sizes 10 and 12 mm diameter for freestream temperatures ranging from 0 to 30°C. He found that the coefficient of heat transfer changes during the melting process inasmuch as the radius of the sphere changes. He also found that the coefficient of heat transfer is lowest for a water temperature of somewhere around 5.5°C.

Merk (2) considered the melting of vertical ice-sheets in water by using an integral momentum method of analysis. He found that the heat-transfer coefficient decreases as the extent of melting increases.

Another work of interest is that of Shenk and Shenkals (3). They studied thermal free convection for the melting of ice spheres in water, for water temperatures between 0 and 10 °C. They obtained flow patterns as well as the local heat-transfer characteristics.

Roberts (4) proposed a simple mathematical model to describe the steady melting of a semi-infinite body of ice, which presents a plane surface transverse to a stream of hot air. This approach can be applied only to the case where the interest is to analyse the melting of a plane surface. Although this model is very simple, it cannot be successfully applied to an irregular surface.

The work of Vanier and Tien (5) is of particular interest to the present work. Their main objective was to investigate the effect of free convection on a more practical geometry and to consider the effect of changing body shape. The authors carried out experiments by melting ice spheres of selected diameters 2,3 and 4 inches for selected free stream temperatures of 5, 7, 11 and 22 °C. An important assumption that was made in this work was that the shape of the body is always that of a sphere. However, in the actual experiments they found that this assumption was violated by the hollowing out of a circular scalloped region at the bottom of the sphere. They attributed this phenomenon to a heavy circulation in the wake of the boundary layer flow around the sphere. They also assumed that the entire heat transfer to ice was accounted for only in the phase change from solid to liquid. This is equivalent to assuming that the entire sphere was at the melting temperature.

Griffin (6) determined the velocity, temperature and concentration distributions near a melting horizontal surface of pure ice in saline water using integral techniques, for free stream water temperatures of 5 and 10°C.

Saitoh (7) studied the heat transfer characteristics around a horizontal ice-cylinder immersed in water, both theoretically and experimentally. A graph between Nusselt number and the angular position was drawn for three typical free stream water temperatures namely 4.6, 5.6 and 7.0°C.

Wilson and Vyas (8) conducted experiments on the velocity profiles near a vertical ice surface melting into fresh water for free stream temperatures ranging from 2 to 7°C. Their results indicated steady state motion upwards when the water temperature is below 4.7°C and downwards when the water temperature is above 7 °C. For intermediate temperatures oscillatory bidirectional flow was observed.

Wilson and Srivastava (9) performed a two dimensional analysis for the heat, mass and momentum transfer during the melting of a horizontal ice-sheet above fresh or saline water flowing at laminar Reynolds number.

Bendell and Gebhart (10) conducted an experimental study of natural convective flow over a vertical ice slab immersed in cold water. The major emphasis of this study was on ambient temperatures in the vicinity of the density extremum of water. They found that the minimum Nusselt number occured at a free stream temperature of 5.6 °C.

From the above review of the literature, the work done in the past can be summarized as follows.

1. The heat transfer characteristics on the melting of ice blocks have been studied in the past, but the study has been restricted to constrained well defined shapes such as spheres, cylinders and vertical and horizontal ice-sheets. However, large icebergs existing in nature have irregular shapes including holes passing through them in some cases. These irregular geometries will have very much more complex flow patterns and melting characteristics. However, some of the melting models such as the

one used in the work of Vanier and Tien (5) can be applied.

2. Most of the previous studies have been of an experimental nature and they have been carried out for particular conditions such as a particular combination of the free stream temperature with the diameter of the sphere and for selected values of the heat transfer coefficients. Therefore, the results obtained from one particular combination need not necessarily apply to the other combination and there arises the necessity to repeat experiments for each combination.

3. In all the studies, the ice blocks were completely submerged. In reality, icebergs float on the water surface, gaining heat by convection partly from air and partly from water. In such a situation, the changing shape affects the stability of an iceberg to the extent that it is continually moving to a small extent in the water. Occasionally as a previously stable shape is modified, it becomes unstable and roll over occurs. Therefore, information obtained from artificially constrained ice blocks will not necessarily apply directly to naturally occuring icebergs.

In view of the above summary, the objectives of the present work can now be broadly defined.

(1). From the literature survey, it is quite clear that any further analysis must consider a more realistic geometry (i.e.) an irregular cross-section. Therefore, a two dimensional shape is considered in this work.

(2). In reality icebergs float on the water surface. Therefore, it is very important to analyse the floating characteristics of an irregular iceberg.

(3). Since melting has a serious impact on certain applications, the effect of melting on the stability of icebergs should be investigated thoroughly. For this reason a method needs to be formulated that will at least crudely model the melting of the iceberg in order that the effects of such melting on the iceberg motion can be studied. Thus, once the algorithms are developed to describe the dynamic behaviour of an iceberg with time varying shape, future work would concentrate on the development of a realistic melting model for a floating iceberg.

2. ANALYSIS

If an infinitely long two dimensional iceberg having irregular cross-section is to float on water, then it will do so in one of the several possible stable equilibrium positions. With the temperature of the iceberg being at the melting point and the temperature of water being higher than that of the iceberg, melting of the iceberg takes place in the portion of the iceberg which is below the water surface. A similar heat transfer mechanism takes place in the portion of the iceberg which is above the water surface. This melting changes the shape of the iceberg as time progress. The change in shape of the iceberg affects the floating stability of the iceberg to the extent that it will be continually moving to a small extent in water. Occassionally a previously stable shape gets modified to the extent that stability is not obtainable by a small motion and the iceberg rolls over to a very different orientation. Broadly speaking, the objective of this work is to model the roll-over. In this work, it was decided to model this phenomenon with the assumption that the only information that needs to be known about the iceberg is its shape defined as co-ordinate points with respect to an arbitrarily chosen Cartesian co-ordinate system. This is accomplished as follows: Suppose that the iceberg is floating in one of the stable positions. Develop a melting model and use it to predict the change in shape of the iceberg. Because of this change in shape, the iceberg will move at least to a small extent in water. Predict the motion of the iceberg due to melting by using the appropriate differential equations describing the translational as well as rotational motion of the iceberg. The motion of the iceberg is predicted by solving numerically the differential equations. The above method of modelling the motion of the iceberg necessitates that the following information be known:

Firstly, all the stable orientations of an iceberg must be known. Since the only information that is known about the iceberg is its shape defined as co-ordinate points with respect to a Cartesian co-ordinate system, it is necessary to develop a method of finding the different orientations at which an irregular iceberg can float in a stable equilibrium position on water.

Secondly, the numerical scheme used to solve the differential equations representing the translational as well as rotational motion of the iceberg must be validated. This can be achieved by using the numerical scheme to predict the motion of the iceberg from an arbitrary initial orientation, for the case of no melting. Then, if the numerical scheme is good, it should bring the iceberg to one of the stable positions.

Thirdly, an algorithm must be developed which can deal with the characteristics of a floating, two dimensional iceberg of irregular cross-section. This algorithm should be capable of handling shapes of a general nature as well as being simple to implement as part of the a computer program. Since an iceberg of infinite length is considered, it is sufficient if the analysis is carried out for unit length.

Before going into greater detail in this section, it is necessary to identify the three phases of this work which are

(1). To develop a method of finding the different stable orientations of a two dimensional irregularly shaped iceberg.

(2). To validate the numerical scheme used to solve the equations of motion.

(3). To develop a melting model and use it to predict the change in shape of the iceberg due to melting. The motion of the iceberg arising due to melting can then be modelled by using the validated numerical scheme.

The three phases of work were carried out by developing three different algorithms. The following sections will describe each of these algorithms.

2.1 STABILITY ANALYSIS :

The objective of this analysis is to develop a method of finding the different stable equilibrium orientations of a two dimensional irregular iceberg. If an iceberg floats on water, then the forces that keep the iceberg in equilibrium are

1. the weight of the iceberg, denoted by W, which acts vertically downward through the centre of gravity of the iceberg and

2. the force of buoyancy, denoted by B, which acts vertically upward, through the centroid of the displaced volume of water.

The determination of the values for B and W as well as the location of the centre of gravity and centre of buoyancy is described in Appendix A. In order that a particular orientation of the iceberg may correspond to a stable equilibrium orientation, it is necessary that the following two conditions must be satisfied. Firstly, the effects of the two forces, buoyancy and weight must be nullified. This condition is called the necessary condition. However, for a particular floating position, even if the effects of these two forces nullify, it does not follow that this position corresponds to a stable equilibrium position, for the reasons explained below.

A particular orientation where the necessary condition is satisfied is said to correspond to a stable equilibrium position, only if, when the iceberg is subjected to a small disturbance which may be translational or rotational, forces or couples are set up that tend to restore the iceberg to its original orientation. This is called the sufficient condition. When the orientation corresponds to an unstable one, any small angular displacement sets up a couple that tends to increase the angular displacement. The stability analysis for a body of symmetrical cross-section can be carried out by using the concept of metacentric height as can be found in many introductory fluid mechanics texts. Such a procedure cannot be applied in this case, because of the irregular cross-sectional shape. Therefore a different procedure based on the same principle used in the metacentric height calculations is developed to suit this problem. This is explained in detail in Chapter 3.

In order that a particular orientation of the iceberg with respect to the waterline surface may correspond to a stable equilibrium position, both the necessary and sufficient conditions must be satisfied. The necessary condition is that the effect of the forces like buoyancy and weight must nullify. The different orientations of the iceberg at which this necessary condition is satisfied may be found by iteration. Then at each of these orientations, it can be checked to see whether the sufficient condition is also satisfied. This is done in Chapter 3. The following explains the method of determining the different orientations at which the necessary condition is satisfied.

Restating the necessary condition mathematically, at those orientations where the necessary condition is satisfied, one must have

$$B=W$$

and

A=0.0, where A is the perpendicular distance between the lines of action of the forces of buoyancy and weight. A free body diagram showing the forces acting on the iceberg is shown in Figure 1.

The objective of this analysis is to find those orientations of the iceberg with respect to the waterline at which B=W and A=0.0. However, for a non-melting iceberg the value of weight does not change. There are two variables that determine the values of B and A. They are

(1). The angular orientation of the iceberg with respect to the waterline, denoted by T

(2). For a particular angular orientation, the position of the centre of gravity of the iceberg with respect to the waterline, denoted by Y.

Since the only information that is known about the iceberg is its shape defined as co-ordinate points with respect to a Cartesian co-ordinate system, it might appear necessary at this stage that the iceberg be analysed for every possible

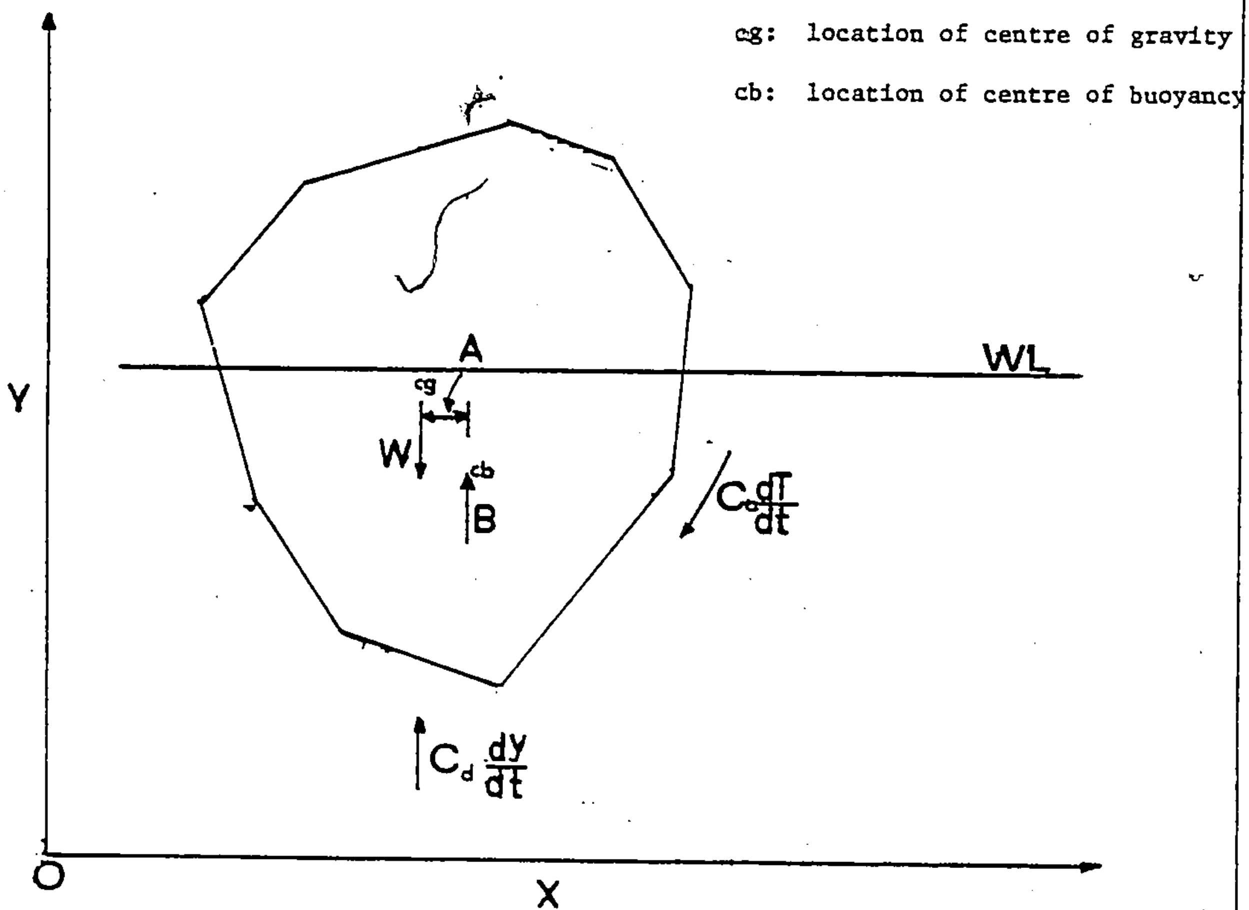


Figure 1. Free Body Diagram of an Irregular Iceberg

orientation of it with respect to the waterline. This would mean that an infinite number of possible combinations of Y and T would require consideration. However, this can be avoided as explained below.

Since there are two variables that determine the values of B and A, it is possible to carry out the analysis, keeping one variable temporarily constant, while varying the other. It is also known that for each angular orientation T, there exists a Y at which B=W. Therefore, for each angular orientation T, the Y at which B=W is found by iteration. Then at each of these orientations, it can be checked to find whether A=0.

When the necessary condition is satisfied, the centre of gravity of the iceberg will be below the waterline. This really means that the orientations of the iceberg that correspond to cases where the centre of gravity is above the waterline need not be analysed. For an angular orientation T, consider that Y at which B=W. Call this Y_C. This value must be negative, because of the sign convention. (This is described in Appendix A). Consider two successive iterations during which the Y value remains higher than Y_C. In the present iteration, the product $(B_0-W)\times(B-W)$ will be positive. B_0 is the value of force of buoyancy during the previous iteration.

Consider two successive iterations during which the Y value remains lower than Y_C. In the present iteration, the product $(B_0-W)\times(B-W)$ will be positive. If the value of Y remains higher than Y_C during the previous iteration and Y value becomes less than Y_C during the present iteration, then the product $(B_0-W)\times(B-W)$ will become negative. The value of Y_C lies between the values of Y during these two successive iterations. The Y_C value can be found by linear interpolation from the previous value. Therefore, if the algorithm is capable of identifying this situation, then this will eliminate the necessity of analysing the iceberg for an infinite number of Y values. This idea is used in this algorithm to find the value of Y at which B=W, for each angular orientation.

Once Y_C is found by the procedure explained above, the value of A is computed as explained in Appendix A. A can have a positive or a negative value or A can even be equal to zero. If A is equal to zero, then this position corresponds to the one at which the necessary condition is satisfied. If the value of A is other than zero, then the sign of A should be checked. If A has the same sign as in the previous iteration, the value of T is incremented by ΔT and the calculations are carried out as before for the new T value. On the other hand, if the sign of A is different from its sign in the previous iteration, then using linear interpolation from the previous value of T, the new value of T is found out. (If the centre of buoyancy lies to the right of the centre of gravity, then the magnitude of A will have a positive sign and if otherwise, it will have a negative sign). This check on the sign of A eliminates the necessity to analyse the shape for an infinite number of T values. The following are the steps of calculation involved in this algorithm. There are two different iteration processes that are involved in this algorithm. The first iteration finds the value of Y_C, for an angular orientation T. The second iteration finds that value of T at which A=0. The first iteration consists of the following steps of calculation.

(1). Compute the value of B, taking the value of Y to be the value corresponding to the initial orientation.

(2). Change Y by adding ΔY to it.

(3). Calculate B again and hence determine the product

$$(B_O-W)\times(B-W)$$

(4). Repeat steps 2 and 3 successively and find that Y at which the product $(B_O-W)\times(B-W)$ is negative.

(5). Use the linear interpolation relation

$$Y_N=Y_O+(Y-Y_O)\times((W-B_O)/(B-B_O))$$

to find the new Y value Y_N. Using this new Y, compute B and check whether B=W. If B is not equal to W, then repeat this step till B=W. Since it is very difficult to get the condition that B is exactly equal to W in a computer, assume that B is exactly equal to W, if the absolute value of the difference between them is less than or equal to 0.0001.

The second iteration process finds the position of the angular orientation T at which A=0.0. This consists of the following steps of calculation.

(1). For the present T value and corresponding Y_C value, compute the value of A as explained in Appendix A.

(2). This A can have a positive or a negative value or can even be equal to zero. If A is equal to zero, then this position corresponds to the orientation at which the necessary condition is satisfied. Then, the value of T is increased by ΔT and the calculations are carried out for the next T value. The calculations are stopped when T reaches a value of 360 degrees.

(3). If A has any other value except zero, then the change of sign of A should be checked. If A has the same sign as it had in the previous iteration, then the value of T is changed by adding ΔT to it and the calculations are carried out as before.

(4). If A has a sign which is different from what it had in the previous iteration, then using linear interpolation

$$T_N = T_0 - (T - T_0) \times (A_0 / (A - A_0))$$

the new value T_N is determined.

(5). At this new value of T_N, the first iteration process is performed to find the value of Y_C.

(6). Once the Y_C value is determined corresponding to this new T_N value, the second iteration is performed.

(7). The calculation procedure is stopped as soon as T reaches a value of 360 degrees.

Thus, the procedure described in this section helps in identifying the different equilibrium orientations of a two dimensional irregular iceberg. This algorithm is cast in the form of a computer program which is given in Appendix C. Provisions were made in the algorithm so that the program would print out the values of A and the corresponding T, whenever B matches W. The information obtained about the values of A and T is used in deciding how many of those equilibrium orientations correspond to stable ones. This is done in Chapter 3.1. This helps in validating the numerical scheme used to solve the differential equations representing the translational as well as rotational motion of the iceberg. This numerical scheme is discussed in the following section.

2.2 DYNAMIC ANALYSIS:

The objectives of this analysis are

(1). to identify the differential equations representing the translational as well as rotational motion of the iceberg

(2). to describe the numerical scheme developed to solve those differential equations

(3). to specify the method used to validate the numerical scheme.

An analysis of forces acting on the iceberg as shown in Figure 1 will result in the following system of equations.

$$M\frac{d^2Y}{dt^2} = W-B-C_d\frac{dY}{dt} \qquad \text{........ (1).}$$

$$I\frac{d^2T}{dt^2} = BA-C_a\frac{dT}{dt} \qquad \text{........ (2).}$$

where C_d and C_a are the damping coefficients. The Y value in equation 1 represents the vertical distance of the centre of gravity of the iceberg from the waterline. The T value represents the angular orientation of the iceberg with respect to the waterline.

A careful inspection of equations (1) and (2) will reveal that both the equations are non-linear. This is because of the fact that B is a non-linear function of both Y and T and that C_d and C_a are dependent on Y and T. Therefore, the principle of superposition does not hold good for these equations. Therefore, detailed consideration of the nature of C_d and C_a is necessary as is described below :

In equation (1), the buoyancy force B represents a force quite similar to the spring force in the case of a linear system. In the case of a linear system, the spring force F is represented by KY, where K is the linear spring constant. The quantity K, by definition is the force per unit displacement in a linear system, represented by dF/dY. By a similar analysis, if B in equation (1) is expressed as $B = \frac{\partial B}{\partial Y} \times Y$, then $\frac{\partial B}{\partial Y}$, will represent a quantity which is similar to a spring constant K in the case of a linear system. The main difference between K and $\frac{\partial B}{\partial Y}$ is that K is independent of Y, whereas, since B may be a non-linear function of Y, $\frac{\partial B}{\partial Y}$ may be dependent on Y. Similarly, $\frac{\partial (BA)}{\partial T}$ represents a quantity which is quite similar to K_θ, the spring constant for angular motion in the case of a linear system.

Therefore, if C_d and C_a are expressed by the following expressions

$$C_d = 2C_1 \sqrt{\frac{\partial B}{\partial Y} \times M} \qquad \ldots(3)$$

$$C_a = 2C_2 \sqrt{\frac{\partial BA}{\partial T} \times I} \qquad \ldots(4)$$

it may be observed that these expressions are quite similar to the expressions in the case of a linear system. Obviously C_1 and C_2 represent the damping ratios in the case of linear system.

To determine approximate values of C_1 and C_2 a small experiment was performed. A schematic of the experimental setup is shown in Figure 2. The objective of the experiment was to determine the displacement versus time curve for small linear motions of a small simulated ice block floating on water. From such a curve, one can determine the value of the damping ratio C_1. Since ice melts quickly in water at room temperature, a piece of wood-block with a small thickness compared with its length and width was attached to a piece of iron-plate and made to simulate an ice block. The dimensions of both the pieces were adjusted such that the weight of the whole piece was equal to that of an ice block of the same volume. This piece was floated in a tank of water. It was made certain that the size of the tank was much larger compared to the size of the piece so that the piece would not touch the side of the tank during the course of the experiment. A displacement transducer was used to record the motion of the piece in water. The output of the transducer was connected to a storage oscilloscope. Sufficient time was allowed so that all the oscillations of the piece in water would die out. Then a small vertical displacement was given to the piece. Then the motion of the piece was recorded in the storage oscilloscope. The displacement versus time curve resembled more or less the displacement versus time curve for a spring, mass damper system, when the spring constant and the damping

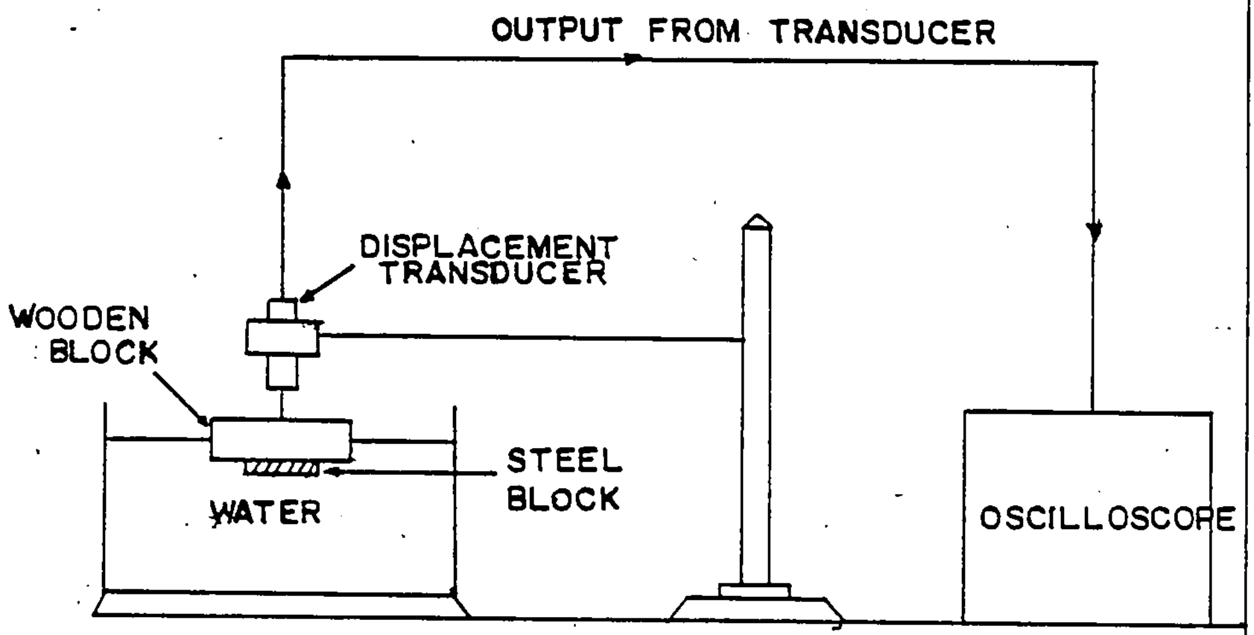


Figure 2. A Schematic of the Experimental Set Up

coefficient are independent of displacement. The consecutive peaks in the displacement were measured from the graduations in the oscilloscope and hence the value of C_1 was determined. The value of C_2 was assumed to be equal to that of C_1. The necessary calculations are shown in Appendix B.

This way of specifying C_d and C_a values are acceptable as long as the iceberg floats on the water surface. But when roll over occurs, it is quite possible that the entire iceberg goes below the water surface for a short time before reaching a stable equilibrium position. When the entire iceberg goes below the water surface, the values of $\frac{\partial B}{\partial Y}$ and $\frac{\partial BA}{\partial T}$ will be zero which corresponds to zero damping force. However, this is not realistic since the iceberg experiences a viscous resisting force for its movements inside water. For this reason, if the entire iceberg goes below the water surface, then the damping force is replaced by means of a drag force calculated as explained below. For this purpose an equivalent cylindrical iceberg is considered. This equivalent cylindrical iceberg is assumed to have the diameter as the maximum distance along the iceberg. Calculated this way, the equivalent cylindrical iceberg will have, for the icebergs considered in this report, a diameter of the order of 150 metres.

The drag force is normally expressed as

$$F_D = 1/2 f_D \, \rho_w \, D_w V_w^2 \qquad \ldots\ldots\ldots\ldots \quad (3a).$$

In equation 3a, f_D is the drag coefficient and is taken to be equal to the drag coefficient for an infinite cylinder. The drag coefficient for an infinite cylinder is 0.48.

$$F_D = 3.6x10^4 V_w^2 \qquad \text{....(3b)}$$

Calculated in a similar fashion, the drag moment for the angular motion is

$$F_{DA} = 1.47x10^{11}\ \omega^2 \qquad \text{....(3c)}$$

Whenever the entire iceberg goes below the water surface, the last term in equations 1 and 2 are replaced by F_D and F_{DA} respectively.

Equations 1 and 2 are interconnected in the sense that solution of one equation is dependent on the solution of the other. This is because of the fact that B is a function of both Y and T. No direct analytical solution of these equations is possible and hence the following solution procedure is used.

Equations 1 and 2 following the transformations $X= \frac{dY}{dt}$ and $Z= \frac{dT}{dt}$, transform into the following system of equations.

$$M \frac{dX}{dt} + B + C_d X = W \qquad \text{....(5a)}$$

$$dY/dt - X = 0 \qquad \text{....(5b)}$$

$$I \frac{dZ}{dt} - BA + C_d Z = 0 \qquad \text{....(6a)}$$

$$\frac{dT}{dt} - Z = 0 \qquad \text{....(6b)}$$

The first derivatives in equations 5 and 6 are replaced by means of a forward difference approximation. The rest of the terms in equation 5a and 6a are evaluated at time step n. The terms X in 5b and Z in 6b are replaced by $\frac{1}{2}(X_{n+1}+X_n)$ and $\frac{1}{2}(Z_{n+1}+Z_n)$ respectively. Under these circumstances the system of equations 5 and 6 reduce to

$$M_n \left(\frac{X_{n+1}-X_n}{\Delta t}\right) + B_n + (C_d)_n X_n = W_n \qquad \text{....(7a)}$$

$$\frac{Y_{n+1}-Y_n}{\Delta t} - \frac{1}{2}(X_{n+1}+X_n) = 0 \qquad \text{....(7b)}$$

$$I_n \frac{(Z_{n+1}-Z_n)}{\Delta t} - (BA)_n + (C_a)_n Z_n = 0 \qquad \text{....(8a)}$$

$$\frac{T_{n+1}-T_n}{\Delta t} - \frac{1}{2}(Z_{n+1}+Z_n) = 0 \qquad \text{....(8b)}$$

The system of equations 7 and 8 are first order accurate because the forward difference approximation is first order accurate. As can be seen from equations 7 and 8, the numerical scheme is self starting.

2.2.1 SOLUTION STRATEGY:

Equations 1 and 2 are second order differential equations. Therefore, four initial conditions (in total) are required to solve these equations.

Specifying the initial orientations of the iceberg with respect to the waterline is equivalent to specifying the values of Y and T at n=0.0 (at time t=0.0). Since two more initial conditions are required, the values of X_n and Z_n are assumed to be zero at n=0.0. This is equivalent to assuming that the iceberg has no initial linear or angular velocity. Since specifying the initial orientation gives the values of W, B, M, I and A at n=0.0, equation 7a can be used to determine the value of X_{n+1}. Then equation 7b can be used to find Y_{n+1}. Similarly equation 8a can be used to find Z_{n+1} which can be used in equation 8b to find T_{n+1}. Likewise, the solution can be marched in time.

Another important calculation necessary for the solution is the determination of values for C_d and C_a. These values are calculated at each time step by equations 3 and 4. This necessitates the evaluation of $\left.\frac{\partial B}{\partial Y}\right|_n$ and $\left.\frac{\partial (BA)}{\partial T}\right|_n$. These derivatives are evaluated by means of the following differencing scheme. That is,

$$\left.\frac{\partial B}{\partial Y}\right|_n = \frac{B_n - B_{n-1}}{Y_n - Y_{n-1}} \qquad \ldots.(9)$$

Similarly,

$$\frac{\partial (BA)}{\partial T} = \frac{(BA)_n - (BA)_{n-1}}{T_n - T_{n-1}} \qquad \ldots.(10)$$

Thus the analysis described in this section helps in modelling the motion of the iceberg, starting from an arbitrary initial orientation. With the assumption that there is no melting, this numerical scheme is used to predict the motion of the iceberg starting from an initial orientation. A computer program for this purpose was written and is given in Appendix C. This modelling of the motion of the iceberg is combined with a melting model to predict the motion of the iceberg arising due to melting. The method used to model the motion of the iceberg arising due to melting and the melting model developed are described in the following section.

2.3 MELTING ANALYSIS :

The objectives of this analysis are

(1). To develop a melting model that can be used to predict the change in shape of the iceberg

(2). To describe a method of modelling the motion of the iceberg arising because of its shape changing due to melting.

In order to predict the motion of the iceberg arising out of melting, the following steps of calculation procedure is used.

(1). The initial orientation of the iceberg is taken to be one of the stable positions assuming that the iceberg has no initial translational or angular velocity.

(2). A time step Δt is selected. It is assumed that the iceberg melts for this time step Δt. The shape of the iceberg at the end of this time step is determined. Because of this melting, the values of weight and buoyancy and moment of inertia change and are recalculated.

(3). For this time Δt , the motion of the iceberg is modelled using the method described in Chapter 2.2

(4). Modelling the motion of the iceberg gives rise to a new orientation and in this new orientation steps 2 and 3 are repeated. Likewise, the analysis is marched in time.

The step 2 described above necessitates that a melting model be developed, which can be used in predicting the shape of the iceberg due to melting. The melting model developed in this work is based on the following assumptions:

(1). The convective heat transfer coefficient and the temperature of the fluid medium surrounding the iceberg are assumed to be uniform. The dimensions of the iceberg are very large. Therefore, the natural convection taking place on its surface is in the turbulent region and the heat transfer coefficient in the case of a turbulent natural convection flow is independent of physical dimensions. Thus, it can be assumed that the heat transfer coefficient around its surface is constant. Since the turbulent boundary layer thickness is very small compared to the dimensions of the iceberg, it can be assumed that the temperature surrounding the iceberg is uniform.

(2). In regions where icebergs occur, the temperature difference driving the phase change is very small and hence sensible heat effects can be considered to be negligibly small compared to latent heat effects. This means that heat conduction into ice is neglected. This corresponds to assuming that the ice is at its fusion temperature throughout.

(3). In this and the succeeding developments, the thermal properties of ice, namely ρ, k and C_p are assumed to be constant and independent of temperature.

Therefore, the heat transferred to the ice by convection through the fluid can be equated to the heat required to melt the ice. Thus, the rate of melting can be expressed as

$$V=-h(T_b-T_f)/\rho\lambda \qquad \text{........ (11).}$$

The rate of melting is the velocity with which the surface of the iceberg will advance into itself per unit time.

This melting model is applied only below the waterline portion. The reason for this is that in regions where icebergs occur, the air temperatures are low and the amount of heat gained from water is greater than the amount of heat gained from air. Hence, melting takes place at a much faster rate below the waterline than above. The rate of melting multiplied by the time step Δt, gives the amount by which the surface of the iceberg will advance during the time Δt. This is called the melt displacement dz.

To calculate the melt displacement the required quantities are

(1). the temperature of ice. (This is assumed to be equal to the fusion temperature of ice at atmospheric pressure which is 0°C)

(2). the bulk temperature of water. This is arbitrarily chosen to be 5.0°C.

(3). the heat transfer coefficient h,

and (4). the density and the latent heat of fusion of ice.

Except for the heat transfer coefficient h, the other quantities can be obtained with ease. To determine the value of the heat transfer coefficient, the following method is used. In

Chapter 2.2, when the drag force was calculated, the diameter of the equivalent cylinder was taken to be 150 meters. This equivalent cylinder replaces the icebergs considered in this report. The value of the heat transfer coefficient is calculated following the equation developed by Raithby and Hollands (15). For this purpose, the Rayleigh number of the flow is estimated. The Rayleigh number is given by

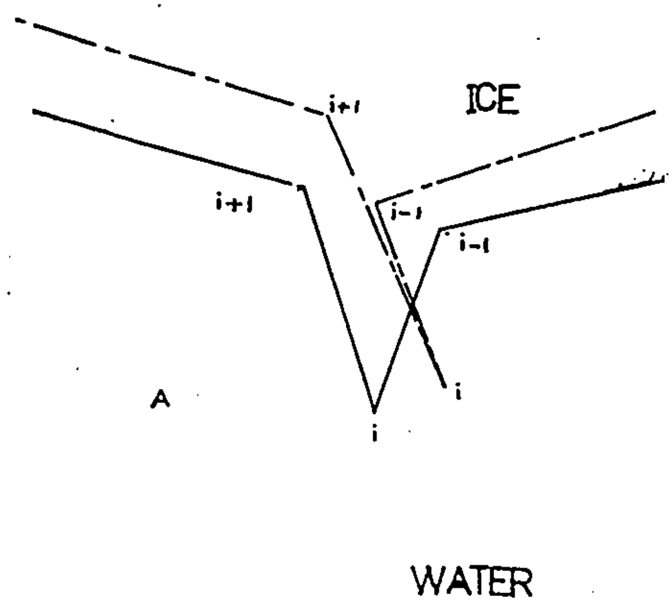
$$Ra = \frac{\beta\ g(T_b - T_i)L^3}{\alpha\nu} \qquad \ldots.(12)$$

Here L is the characteristic length and is taken to be the diameter of the equivalent cylinder. For a T_b=5.0°C, the Rayleigh number is 5.06×10^{17}. Obviously, this Rayleigh number range corresponds to turbulent regime of the natural convection flow. For the turbulent regime, in the case of a cylinder, Raithby and Hollands(15) developed the following equation relating the Nusselt number and the Rayleigh number

$$Nu = 0.101\ P_r^{0,084}\ .\ Ra^{1/3} \qquad \ldots.(13)$$

In this equation, the length dimensions in Nusselt number and Rayleigh number cancel each other leaving the heat transfer coefficient independent of physical size. Using the above equation the Nusselt number is calculated. From the Nusselt number the average value of the heat transfer coefficient is calculated, assuming the characteristic dimension to be the diameter of the equivalent cylinder. The heat transfer coefficient thus obtained is increased by a factor of 6 in order to account for all the complexities involved in the natural convection flow, because of the irregular cross-section. This corresponds to a heat transfer coefficient of approximately 2000 W/m°K

Once the melt displacement is calculated as explained above, this melting procedure is applied to the portion of the iceberg which is below the water surface. This is done as follows. Each co-ordinate point describing the iceberg which lies below the water surface is given the melt displacement dz into the iceberg. This displacement dz is given in a direction perpendicular to the line joining the point under consideration and the next point with a direction into the iceberg. The new co-ordinate values of the points below the waterline are thus obtained. Once this type of calculation is carried out for all the points below the waterline, the new shape defining the iceberg is obtained. However, this procedure has the following drawback. Consider a sharp corner in the iceberg such as the one shown in Figure 3. Under melting, such a portion of an iceberg will undergo shape changes as shown in Figure 3. In Figure 3B, there is a cross-over between the points defining the shape of the iceberg and therefore an unrealistic shape is produced. The



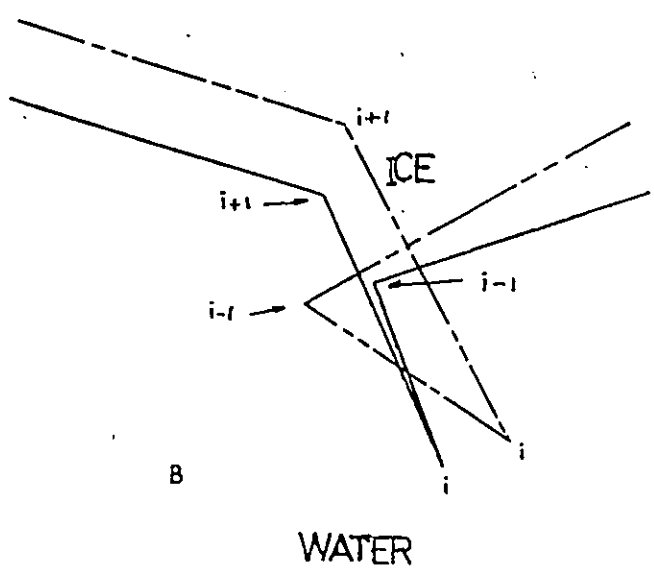


Figure 3. Illustration of the Cross-over Problem

algorithm developed in this work will fail under such a cross-over. Therefore, a good algorithm must identify such situations and hence prevent cross-overs. This is accomplished as follows. A right handed rectangular Cartesian co-ordinate system with its origin at point i and positive x-axis along the points joining i and i-1 is attached. With respect to this co-ordinate system, the point i+1 will always lie in the first quadrant. Therefore, the angle subtended by the line joining the point i+1 and the origin with the x-axis of this co-ordinate system will always be less than or equal to 90 degrees. If this angle becomes less than or equal 45 degrees during the calculation, then the co-ordinates of point i will be replaced by the average of the co-ordinate points between i-1 and i+1. This is equivalent to assuming that the point i lies in the midpoint of the line joining points i-1 and i+1.

Thus the analysis described in this section helps in modelling the motion of the iceberg arising due to melting. A computer program for this purpose is given in Appendix C.

The following summarizes the three phases of work described in this Chapter.

(1). A procedure to determine the different equilibrium orientations of a two dimensional irregular iceberg is described.

(2). A numerical scheme is developed to solve the differential equations representing the translational as well as rotational motion of the iceberg.

(3). A method is described to model the motion of the iceberg arising from melting. A simple melting model is developed to predict the change in shape of the iceberg due to melting.

3. RESULTS AND DISCUSSION

3.1 RESULTS OF STABILITY ANALYSIS:

In Chapter 2.1, it was established as to how one can find out the various orientations of a two dimensional irregularly shaped iceberg at which the necessary condition is satisfied. The objective of this analysis was to find those orientations of the iceberg at which the weight and buoyancy forces match each other under the condition that the value of the moment arm A is zero. This was done in a systematic manner. For each angular orientation T, the value of Y at which the weight matches the force of buoyancy was found out by iteration. Starting from a particular angular orientation, the iceberg was rotated through 2π radians. A method by which the necessity to analyse the iceberg for the infinite number of Y and T combinations can be avoided was also indicated. This algorithm was cast in the form of a computer program which is given in Appendix C. This computer program needs the x and y co-ordinates describing the shape of the iceberg and an initial orientation with respect to the waterline. Provisions were made in the algorithm so that the program would print out the values of A and T whenever W matches B. If a graph is drawn taking the values of T along the abscissa and the values of A along the ordinate, then every point in this graph will correspond to that orientation of the iceberg with respect to the waterline surface, at which the weight and buoyancy forces match each other. Points on the graph that lie on the abscissa correspond to situations in which the effects of weight W and the force of buoyancy B nullify. In other words, these points (points on the abscissa) correspond to cases in which the necessary condition is satisfied. Some of these points may correspond to stable equilibrium orientations but others may be unstable. If any equilibrium orientation corresponds to a stable one, then at that orientation, a small rotation of the iceberg must shift the centre of buoyancy in such a way that this shift in the centre of buoyancy must produce an effect that opposes the rotation of the iceberg and the combination must produce an effect such that it will bring the iceberg to its original orientation.

In simpler words, if an iceberg floating in stable equilibrium position is given a small rotation clockwise, then the centre of buoyancy should shift to the right of centre of gravity and similarly, if an anticlockwise rotation is given, then the centre of buoyancy should shift to the left of centre of gravity.

In this work, A is assumed to be positive if the centre of buoyancy lies to the right of centre of gravity and negative, if it is to the left of centre of gravity and T is assumed to be positive if anticlockwise and negative if clockwise. If a small anticlockwise rotation (a positive change in T) is given to an iceberg at a stable position, then from the above rule and sign convention, it is quite clear that the value of A will decrease (a negative change in A). On the other hand, if a small clockwise rotation (a negative change in T) is given to an iceberg at a stable position, then from the above rule and sign convention, it is quite clear that the value of A will increase (a positive change in A). Therefore, it can be concluded that at a stable position, a positive change in T produces a negative change in A and vice versa. This suggests that the value of the derivative dA/dT should be negative at an equilibrium position for that one to be a stable one .

If dA/dT is positive at an equilibrium position, from the sign convention, it is quite clear that the effect of a small angular disturbance (a small change in T) is to shift the centre of buoyancy in such a way as to set up a couple that tends to increase the angular disturbance. Therefore, obviously, if dA/dT is positive at an equilibrium position, then that corresponds to an unstable equilibrium orientation.

Three shapes were analysed in this work and they are shown in Figures 4a,4b and 4c. Figure 5 shows the A versus T graph for the typical shape shown in Figure 4a. It has totally six equilibrium orientations, since the graph has six points on

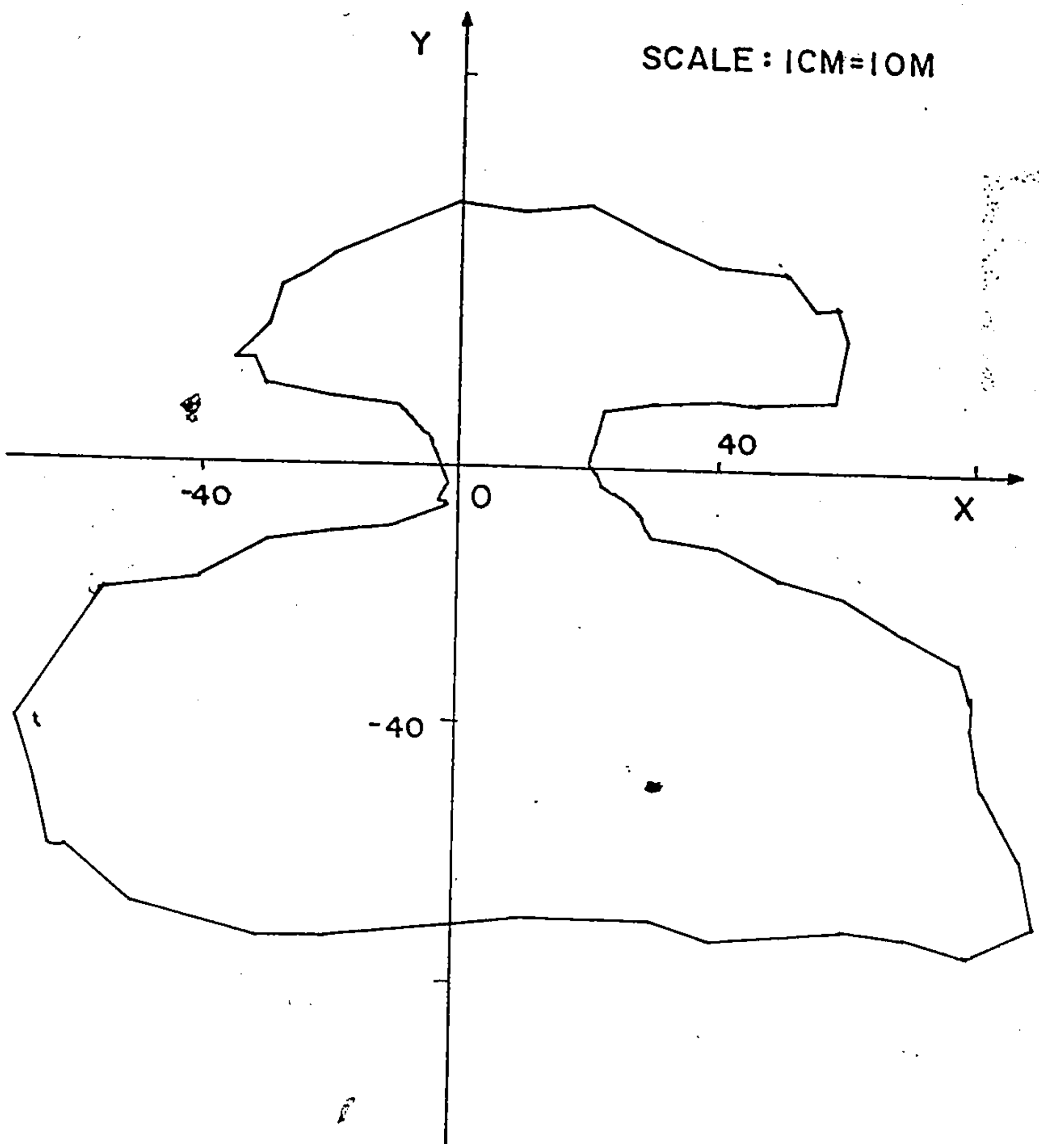


Figure 4a. The Typical Shape as Defined by Co-ordinates in Table 3.

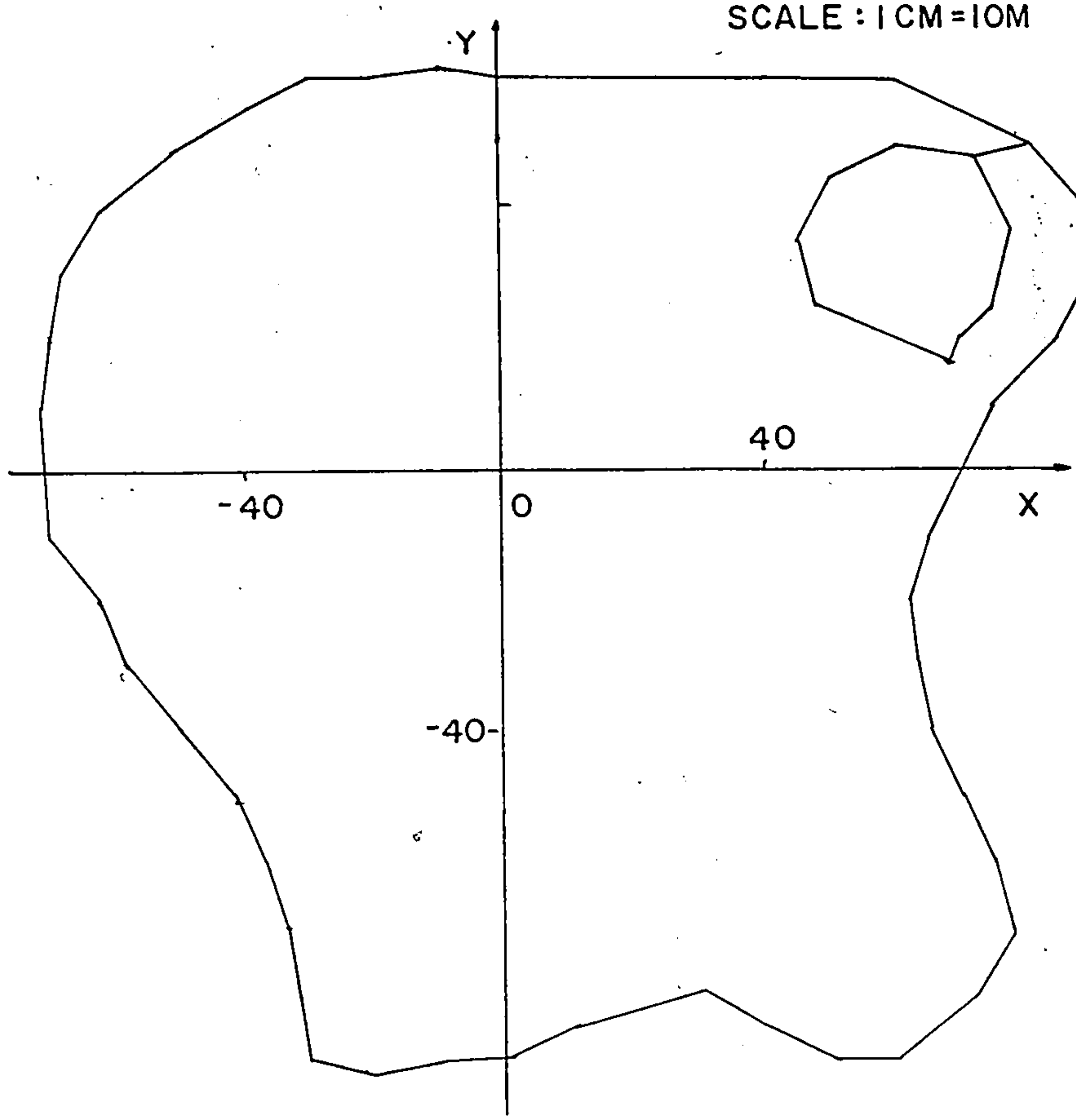


Figure 4b. The Irregular Shape with a Hole as Defined by Co-ordinates in Table 4

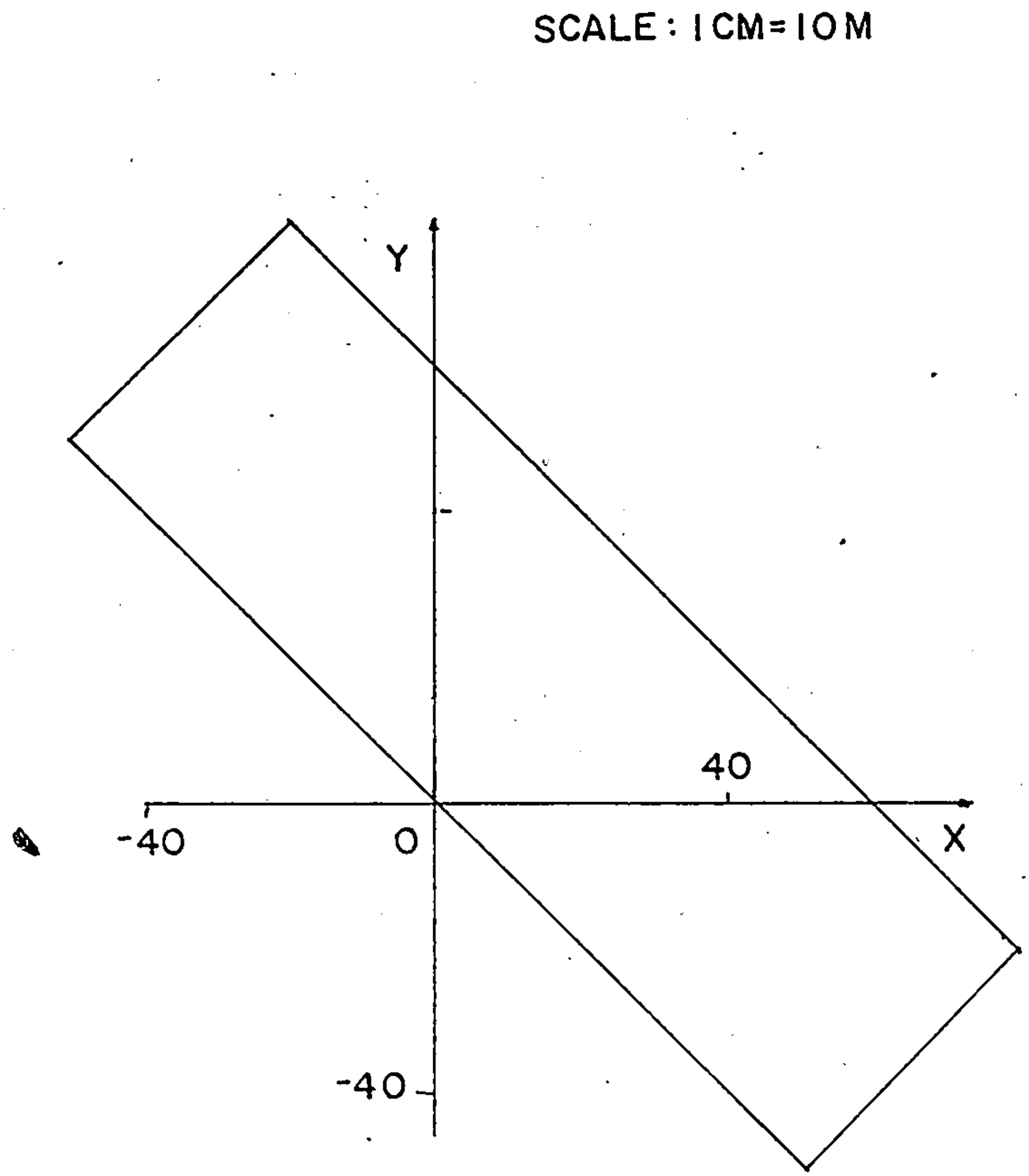


Figure 4c. The Rectangular Shape as Defined by the Co-ordinates in Table 5

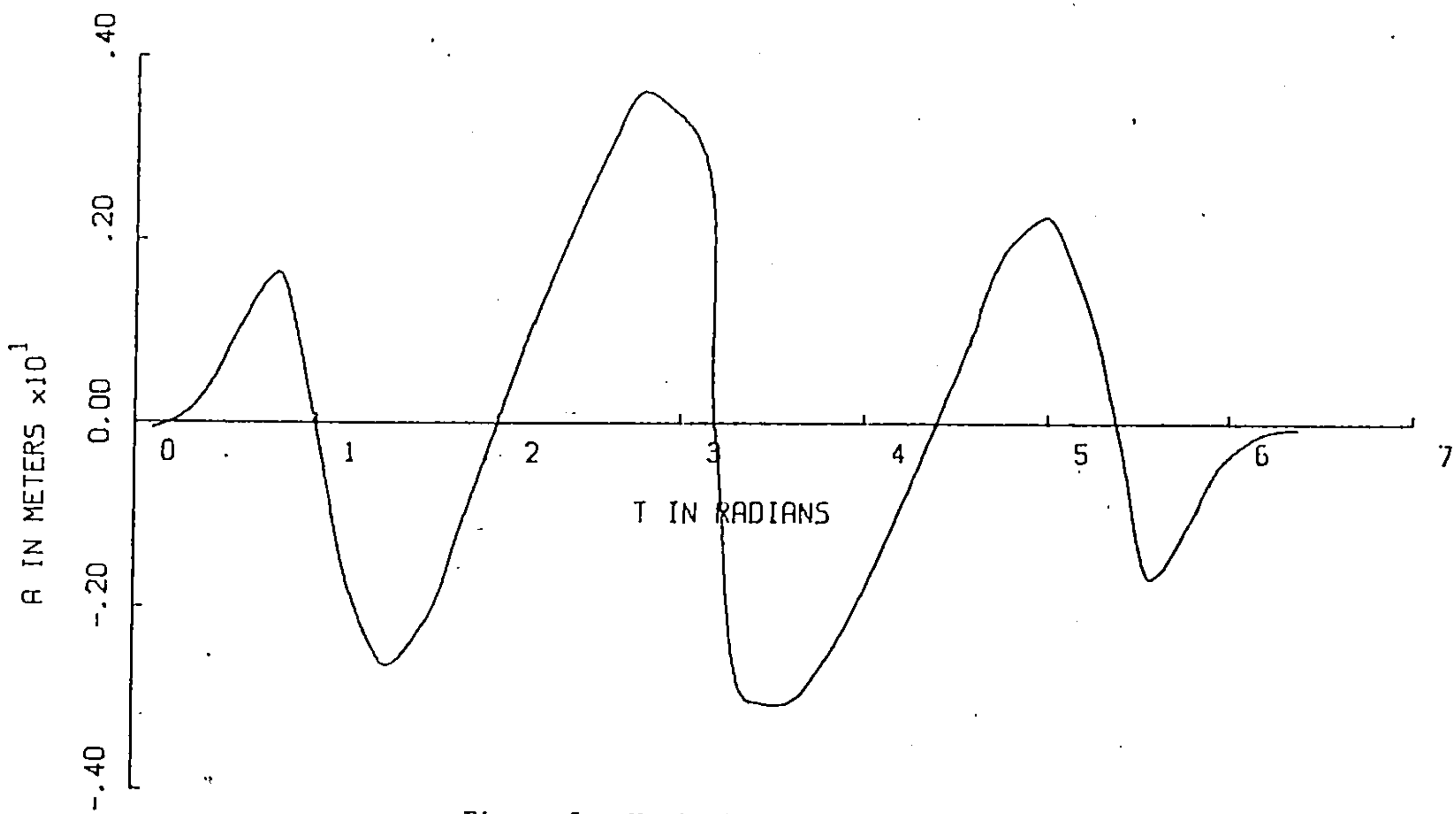


Figure 5. Variation of Moment Arm with Angular Orientation for the Typical Shape

T = 0.183448 Y=-51.6692

dA/dT = 0.817860 SCALE 1:35

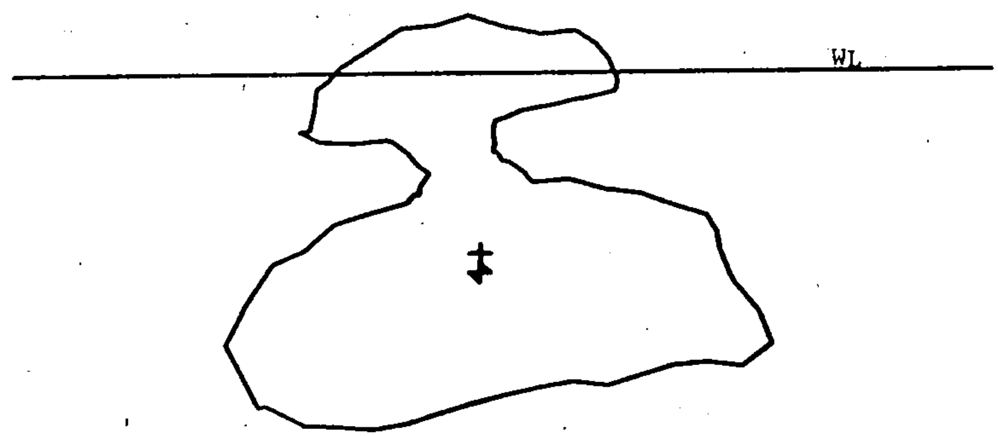


Figure 5a. Unstable Equilibrium Orientation of the Typical Shape

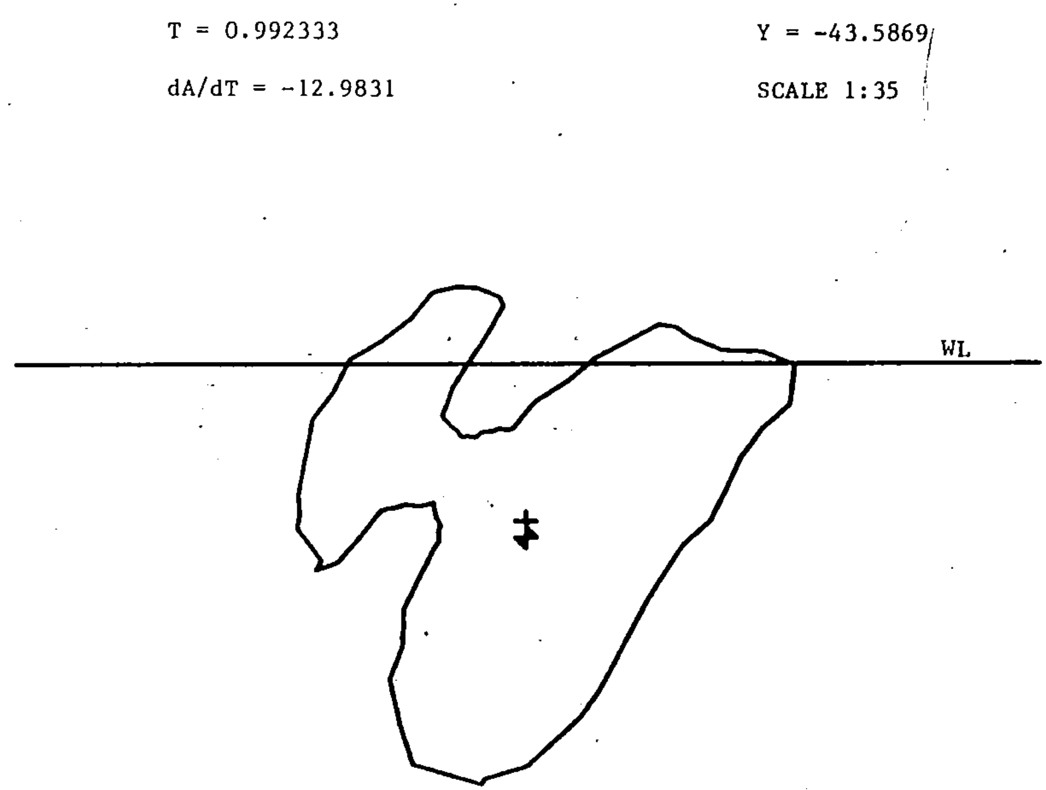


Figure 5b. Stable Equilibrium Orientation of the Typical Shape

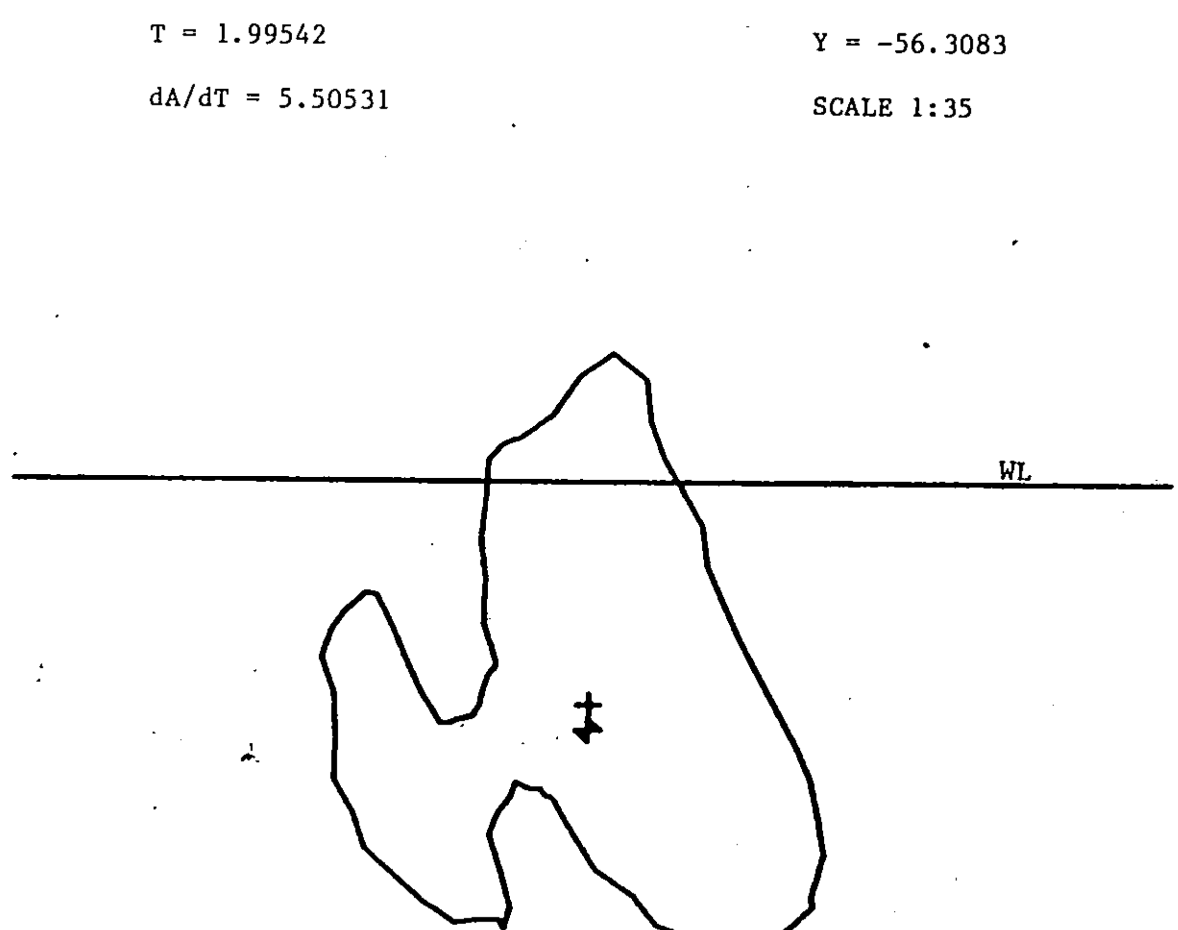


Figure 5c. Unstable Equilibrium Orientation of the Typical Shape

T = 3.18985 Y = -36.5131

dA/dT = -23.7409 SCALE 1:35

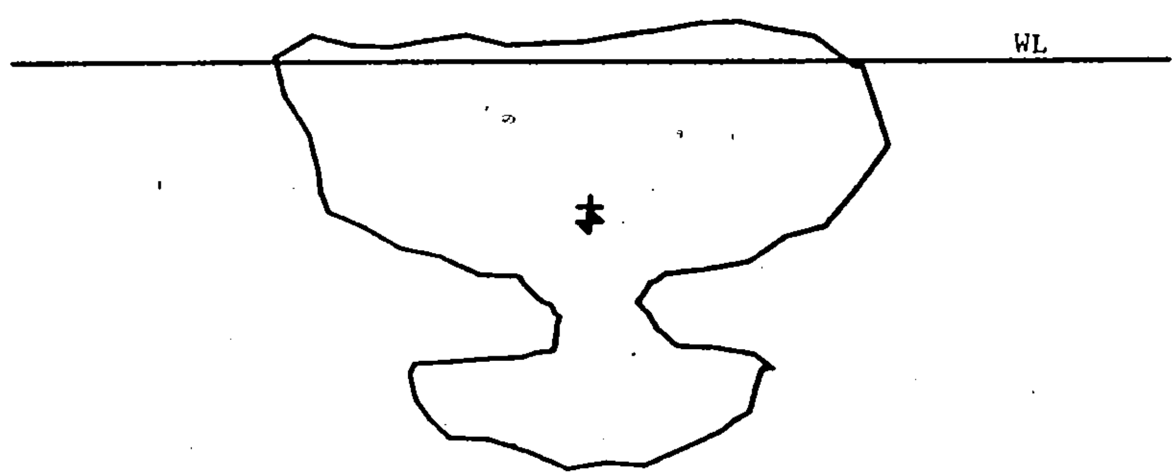


Figure 5d. Stable Equilibrium Orientation of the Typical Shape

T = 4.39425 Y = -55.2435

dA/dT = 4.79360 SCALE 1:35

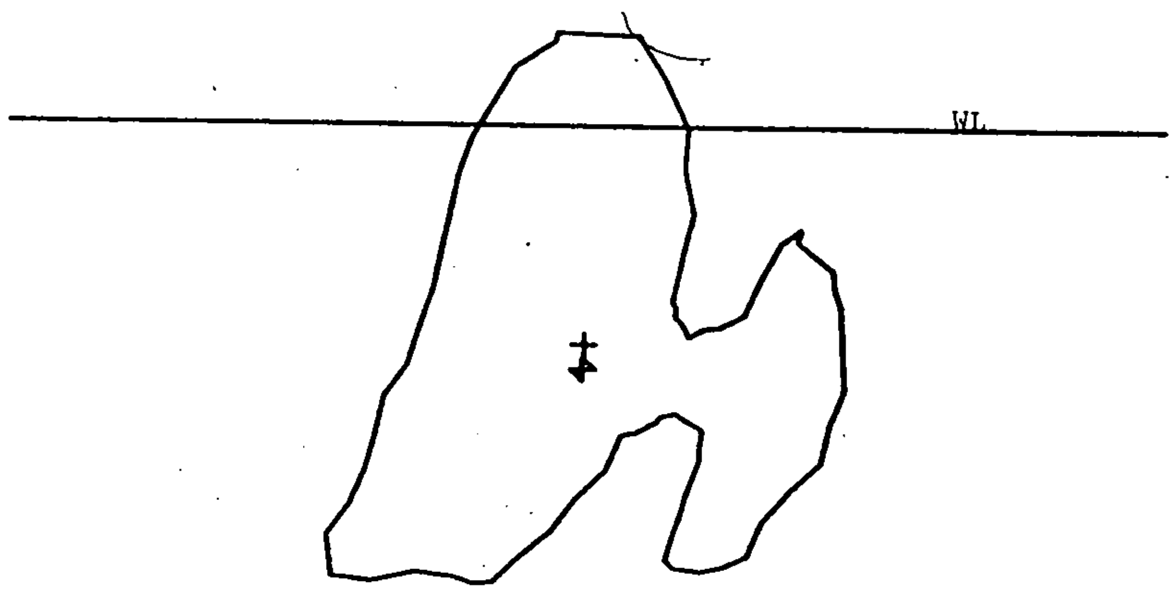


Figure 5e. Unstable Equilibrium Orientation of the Typical Shape

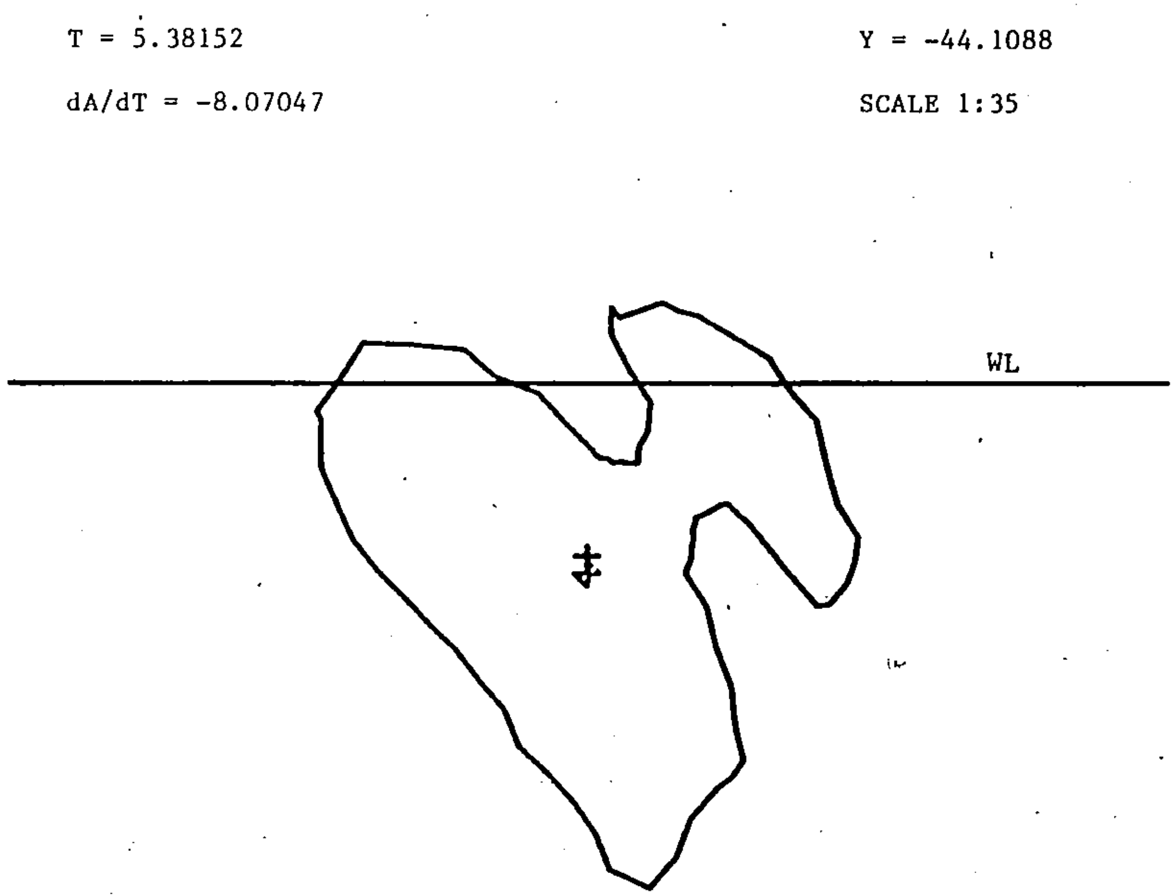


Figure 5f. Stable Equilibrium Orientation of the Typical Shape

the abscissa. The curve starts from a T value of 0.1 rad with a negative A value. It crosses the T axis at T=0.183 rad. At this point, the slope of the A versus T is positive, which means that it corresponds to an unstable equilibrium orientation. This orientation is shown Figure 5a.

The curve crosses the T axis again at T=0.992 rad and at this point, it has a negative slope. This corresponds to a stable equilibrium orientation and this is shown in Figure 5b. The third cross-over occurs at T=1.995 rad and the slope of the A versus T curve at this point is positive, which means that this orientation corresponds to an unstable position. This is shown in Figure 5c.

The fourth cross-over is at T=3.189 rad and the curve has a negative slope. This means that this orientation corresponds to a stable one and this is shown in Figure 5d. The fifth cross-over is at T=4.394 rad and the curve has a positive slope, which means that this orientation is an unstable one and is shown in Figure 5e.

The last cross-over is at T=5.38 rad and the curve has a negative slope. This suggests that this orientation corresponds to a stable one and is shown in Figure 5f.

From the graph shown in Figure 5, it can also be observed that the value of dA/dT is successively positive or negative, as T increases from 0 to 2π radians. The changes in sign of dA/dT correspond to cases of unstable and stable orientations. If dA/dT were to become zero for a particular shape, then that orientation would correspond to the case of a neutral equilibrium position. For example, the value of dA/dT will always be zero, if an iceberg with circular cross-section were considered.

One more observation that can be made from the graph is that for any shape, the number of stable equilibrium orientations must be equal to the number of unstable equilibrium orientations, unless an intervening neutral equilibrium position occurs. The first and the last point in the A versus T curve must be the same. This is because the first and the final orientation of the iceberg with respect to the waterline are the same, since the iceberg is rotated through 2π radians. If, for example, the initial point has a positive A value, then any cross-over of the T axis, of the A versus T graph must have a corresponding cross-over back so that the final point will have a positive A-value. The same argument holds good if the initial point has a negative A value. This really means that number of cross-overs of A versus T curve must always be even.

It is established previously that each cross-over corresponds to either a stable or unstable orientation. A cross-over where the A value changes from positive to negative corresponds to a stable orientation whereas a negative to positive cross-over corresponds to an unstable orientation. Since the number of cross-over should always be even, it can be said that the number of positive to negative cross-overs must be equal to the number of negative to positive cross-over. This really means that the number of stable positions must be equal to the number of unstable positions.

However, this conclusion is not true, if an intervening neutral equilibrium position occurs. At a neutral equilibrium position dA/dT is zero. This means that the A versus T graph will be tangent to the abscissa. A cross-over of the T axis, of the A versus T graph can take place through a neutral equilibrium position.

The A versus T graph for the irregular shape with a hole is shown in Figure 6. Figures 6a to 6f show the equilibrium orientations of the shape. As can be observed from the graph, this shape has three stable equilibrium orientations and three unstable ones. The stable ones are shown in Figures 6b, 6d and 6f. Here also it can be observed that stable and unstable orientations occur one after the other as T is increased from 0 to 2π. The reason for analysing this shape is to show that the

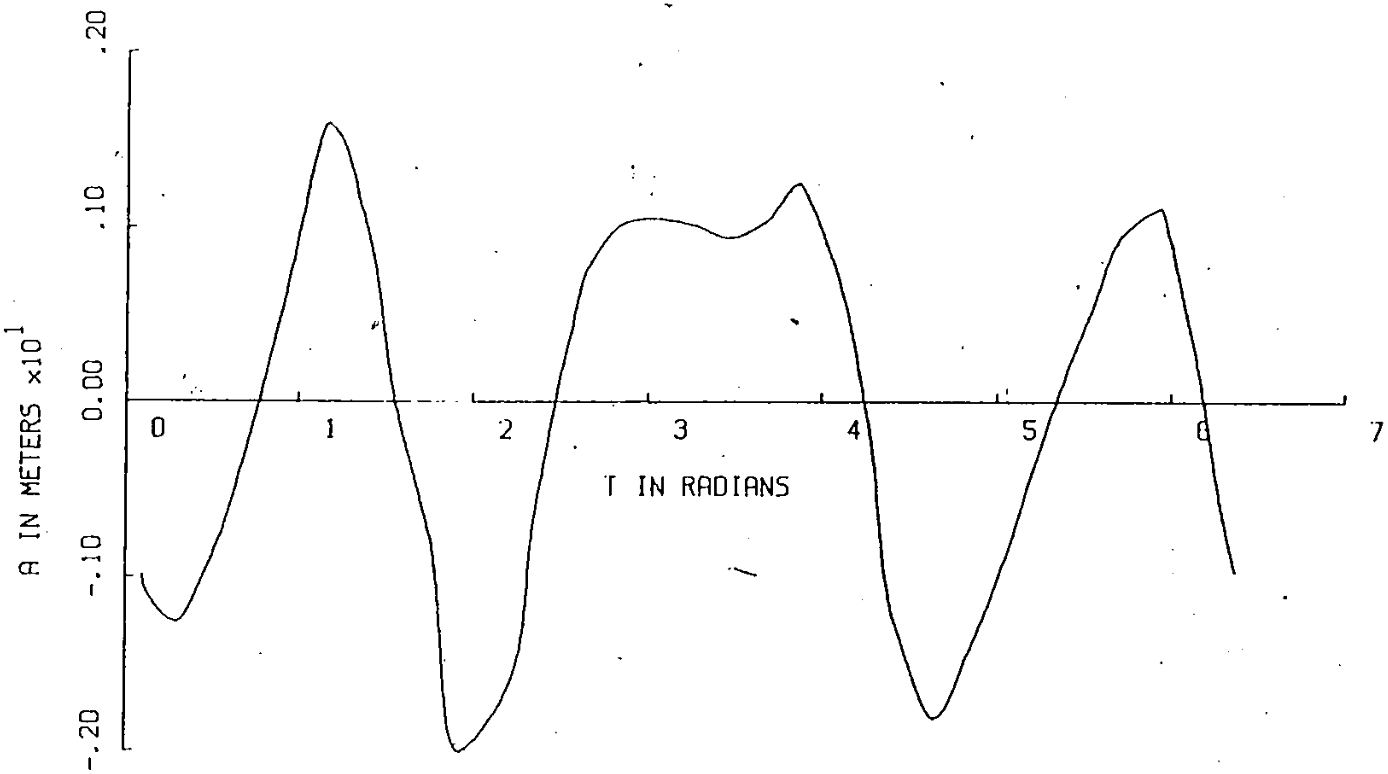


Figure 6. Variation of Moment Arm with Angular Orientation for the Irregular Shape with a Hole

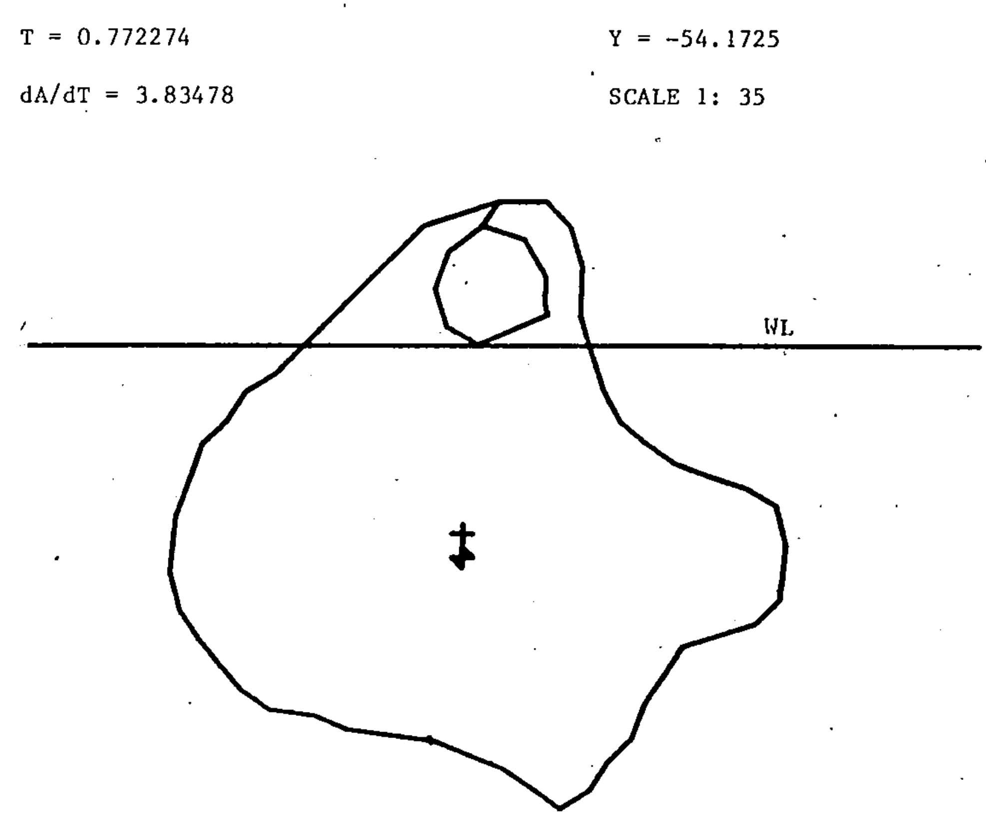


Figure 6a. Unstable Equilibrium Orientation of the Irregular Shape with a Hole

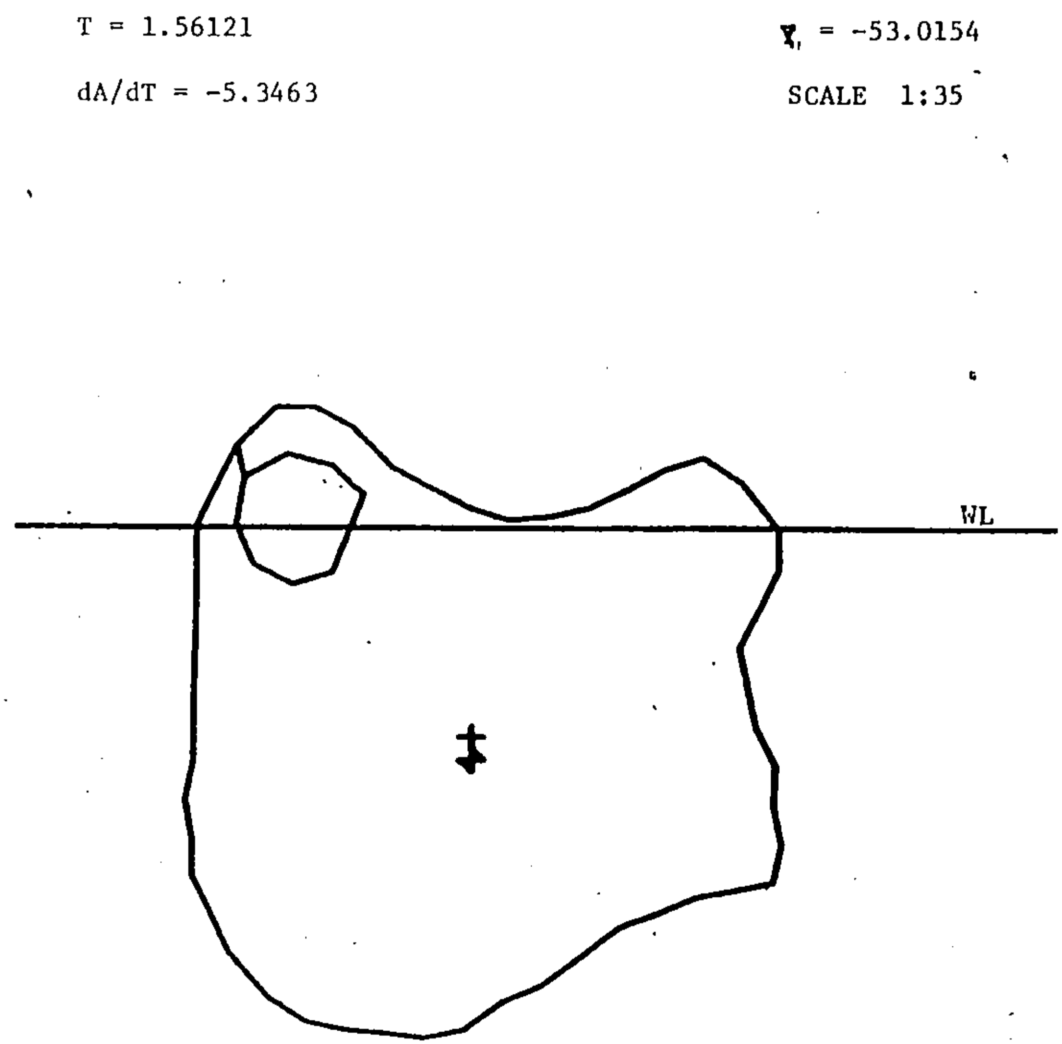


Figure 6b. Stable Equilibrium Orientation of the Irregular Shape with a Hole

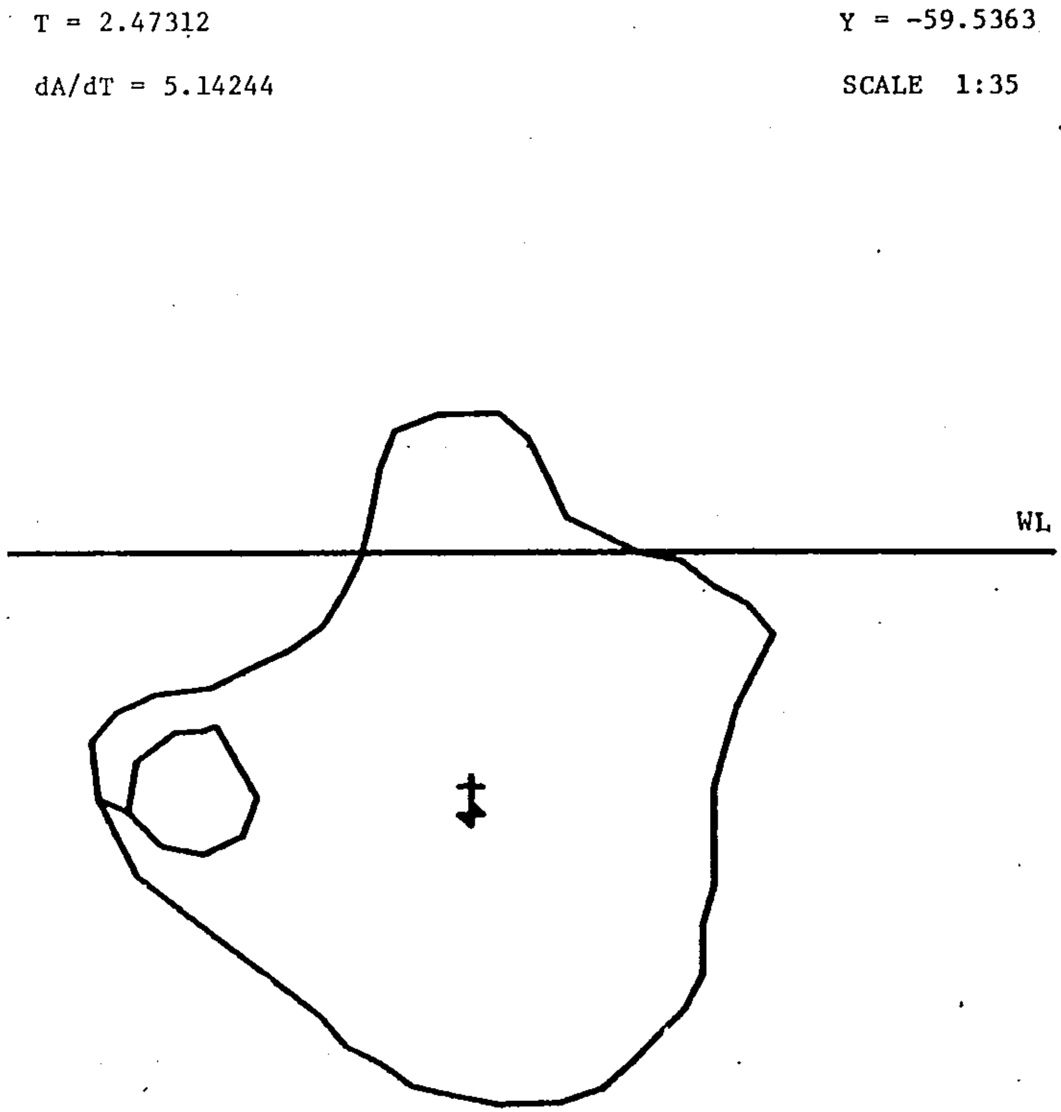


Figure 6c. Unstable Equilibrium Orientation of the Irregular Shape with a Hole

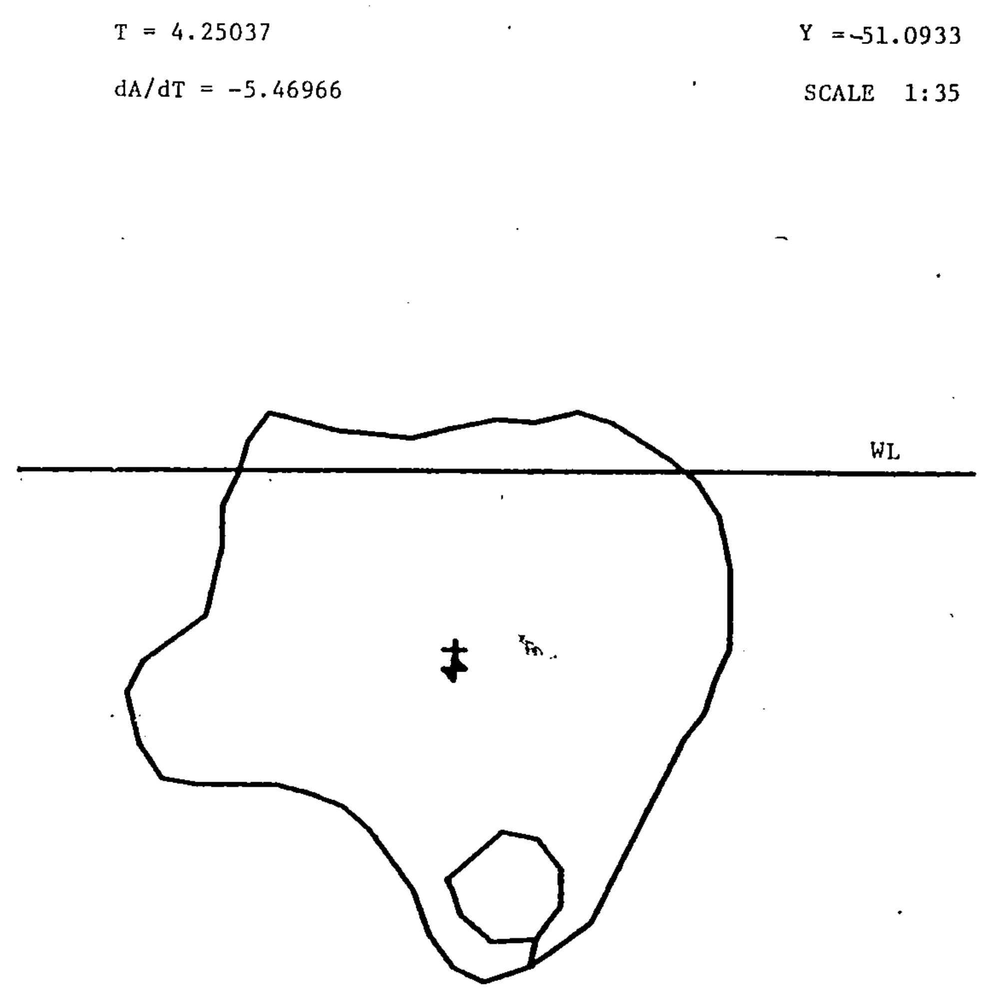


Figure 6d. Stable Equilibrium Orientation of the Irregular Shape with a Hole

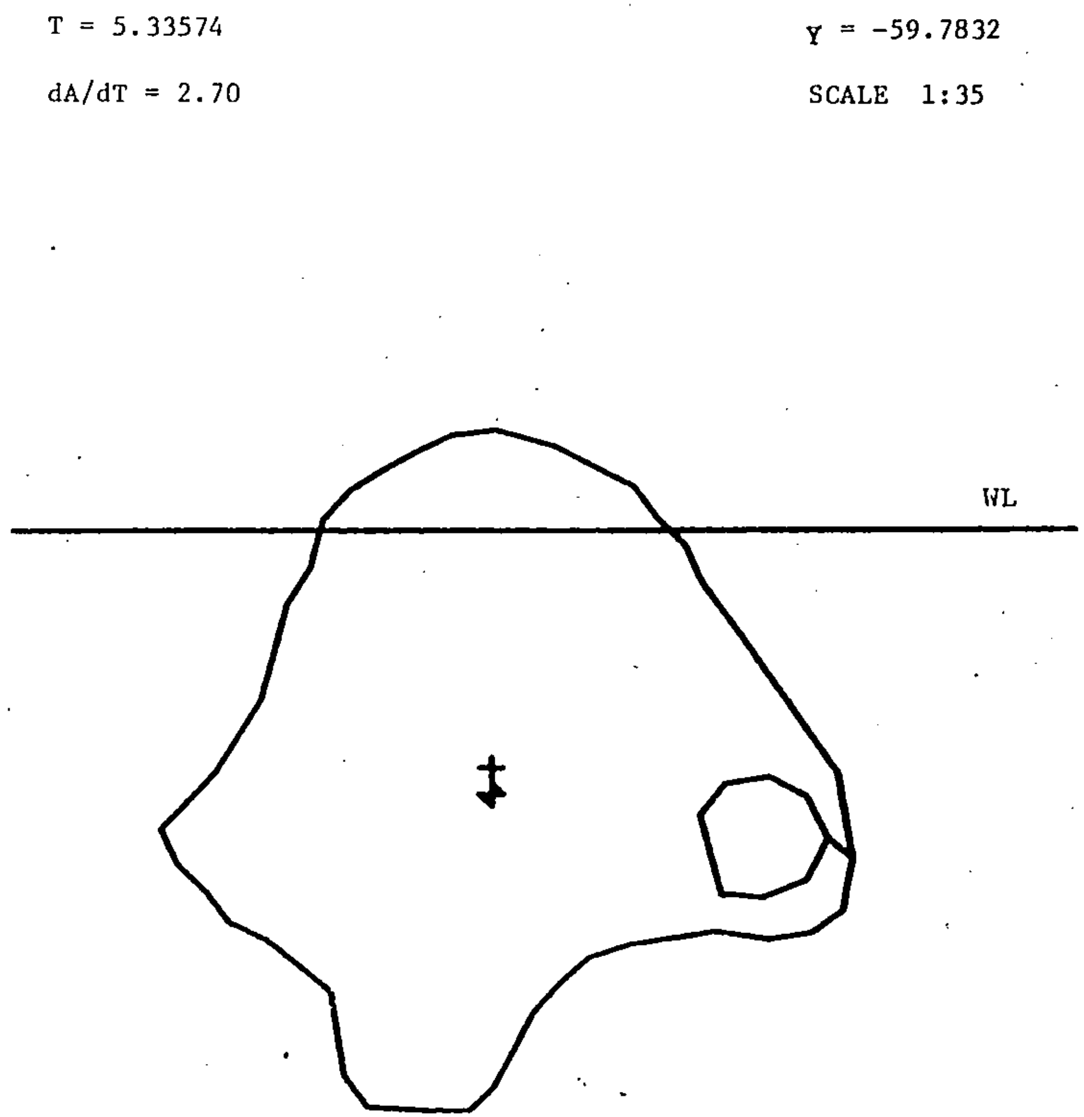


Figure 6e. Unstable Equilibrium Orientation of the Irregular Shape with a Hole

T = 6.17634

dA/dT = -6.80784

ψ = -57.1651

SCALE 1:35

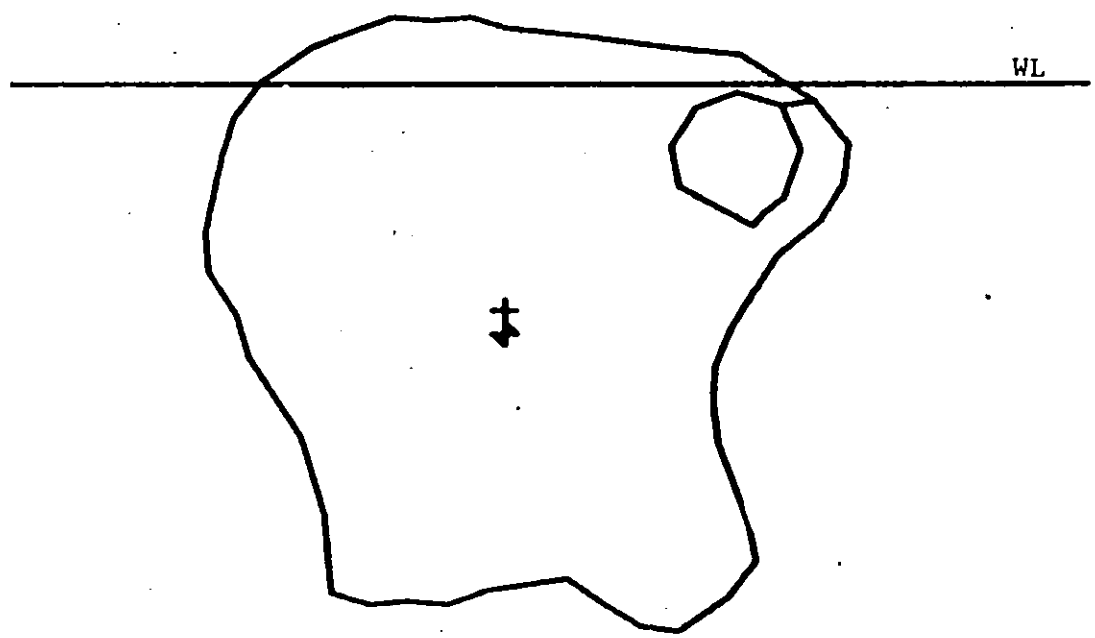


Figure 6f. Stable Equilibrium Orientation of the Irregular Shape with a Hole

algorithm developed in this work can be applied to a shape having a hole in it.

Finally, the same algorithm is applied to a rectangular shape. The reasons for this are several. Firstly, it can be checked to see whether the algorithm developed in this work holds good for a symmetrical shape. Secondly, for a rectangular shape, the stable equilibrium orientations are known. Therefore, it can be checked whether the procedure developed in this work is correct or not. Thirdly, the two orientations of the rectangle are equally stable. Therefore, those points on the A versus T graph that correspond to these orientations must have the same value for the slope. The A versus T graph for this rectangular shape is shown in Figure 7. The graph shows that there are two stable equilibrium positions. The four equilibrium orientations of the rectangle are shown in Figures 7a to 7d. The value of dA/dT for the two stable positions are the same, which indicates that the two positions are equally stable.

The information as to how stable one orientation is compared to the other for a particular shape can be obtained from the magnitude of dA/dT. This is because the value of dA/dT gives the information as to how A is going to change for a small change in T. For this purpose, the value of dA/dT is evaluated by means of a backward difference formula.

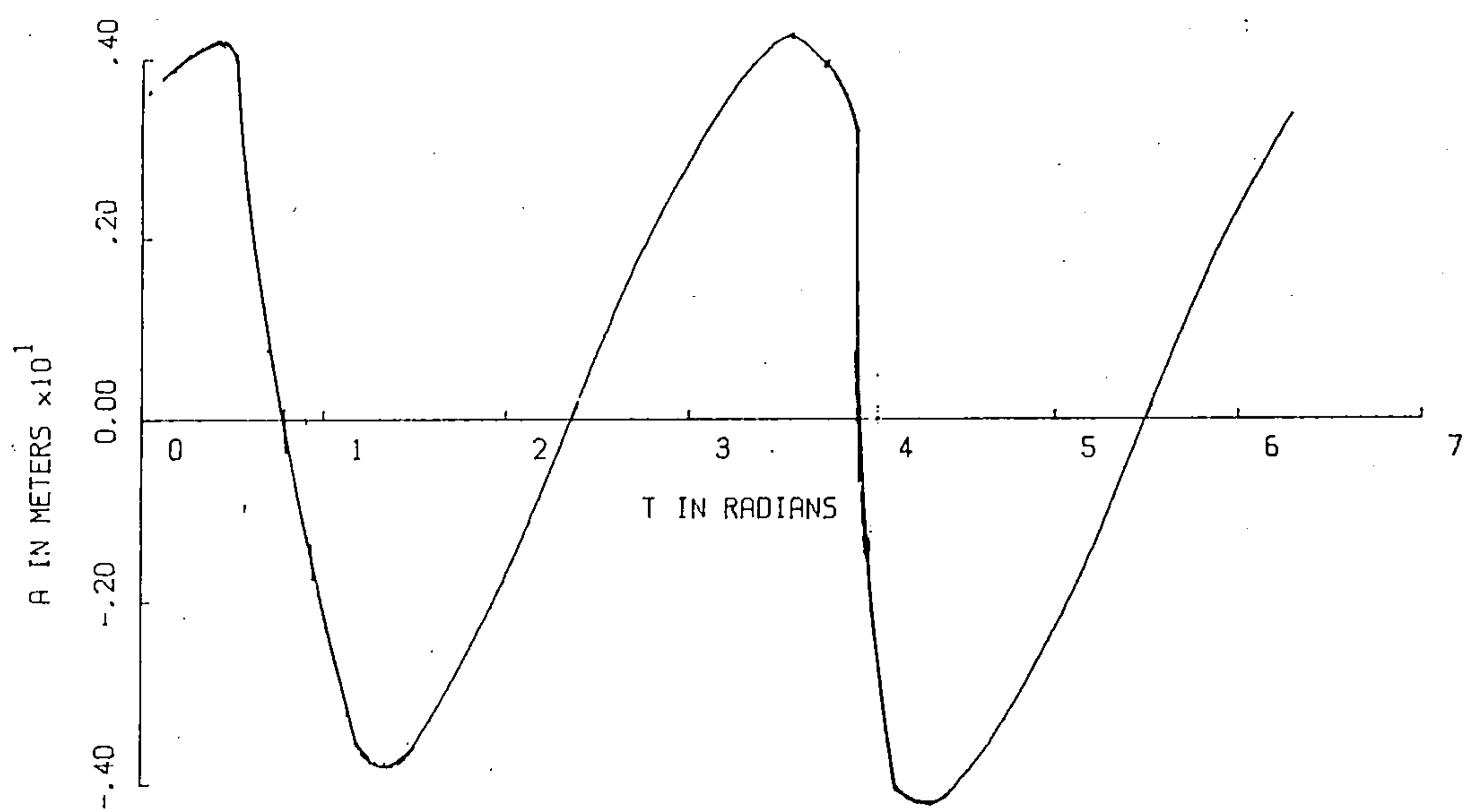


Figure 7. Variation of Moment Arm with Angular Orientation for the Rectangular Shape

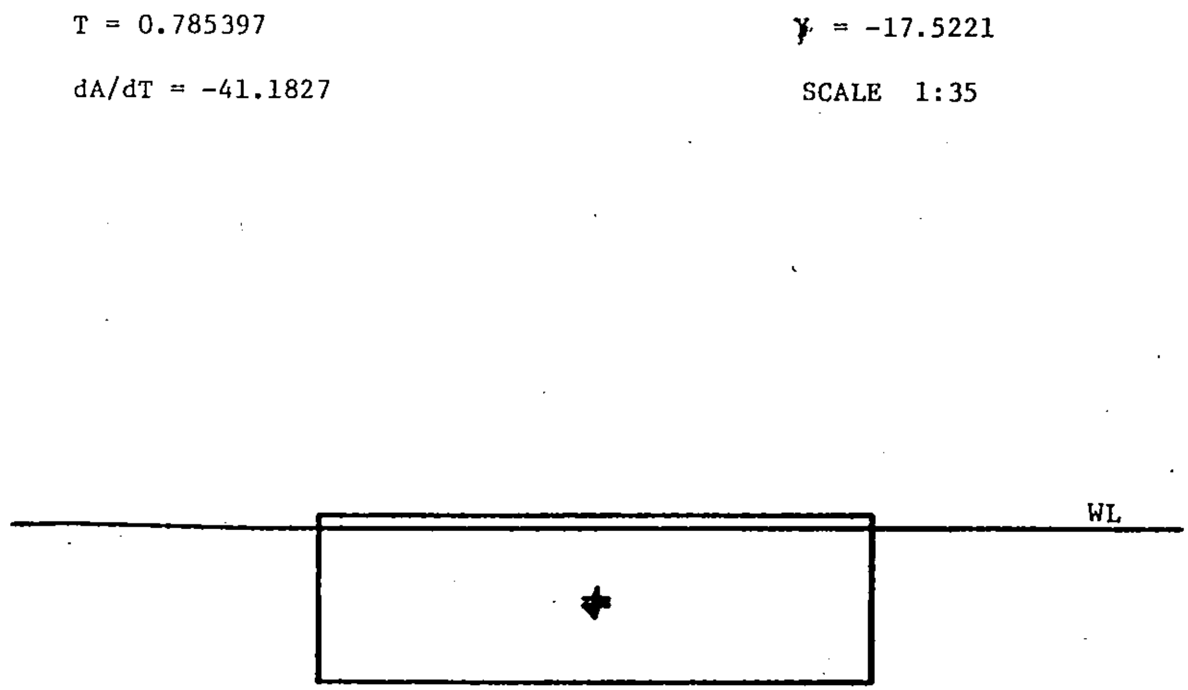


Figure 7a. Stable Equilibrium Orientation of the Rectangular Shape

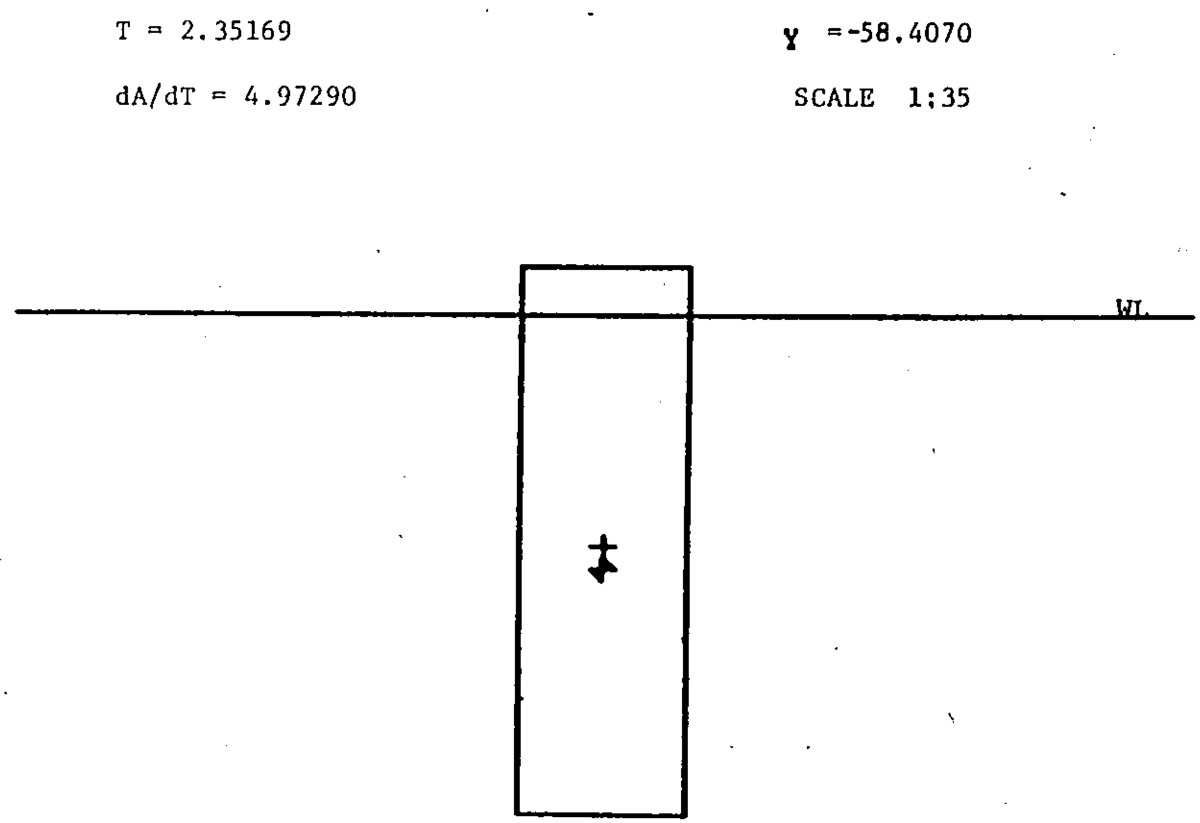


Figure 7b. Unstable Equilibrium Orientation of the Rectangular Shape

T = 3.92699

dA/dT = -41.1865

Y = -17.5221

SCALE 1:35

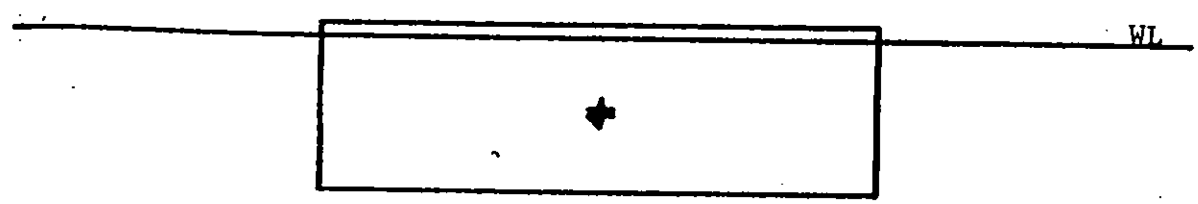


Figure 7c. Stable Equilibrium Orientation of the Rectangular Shape

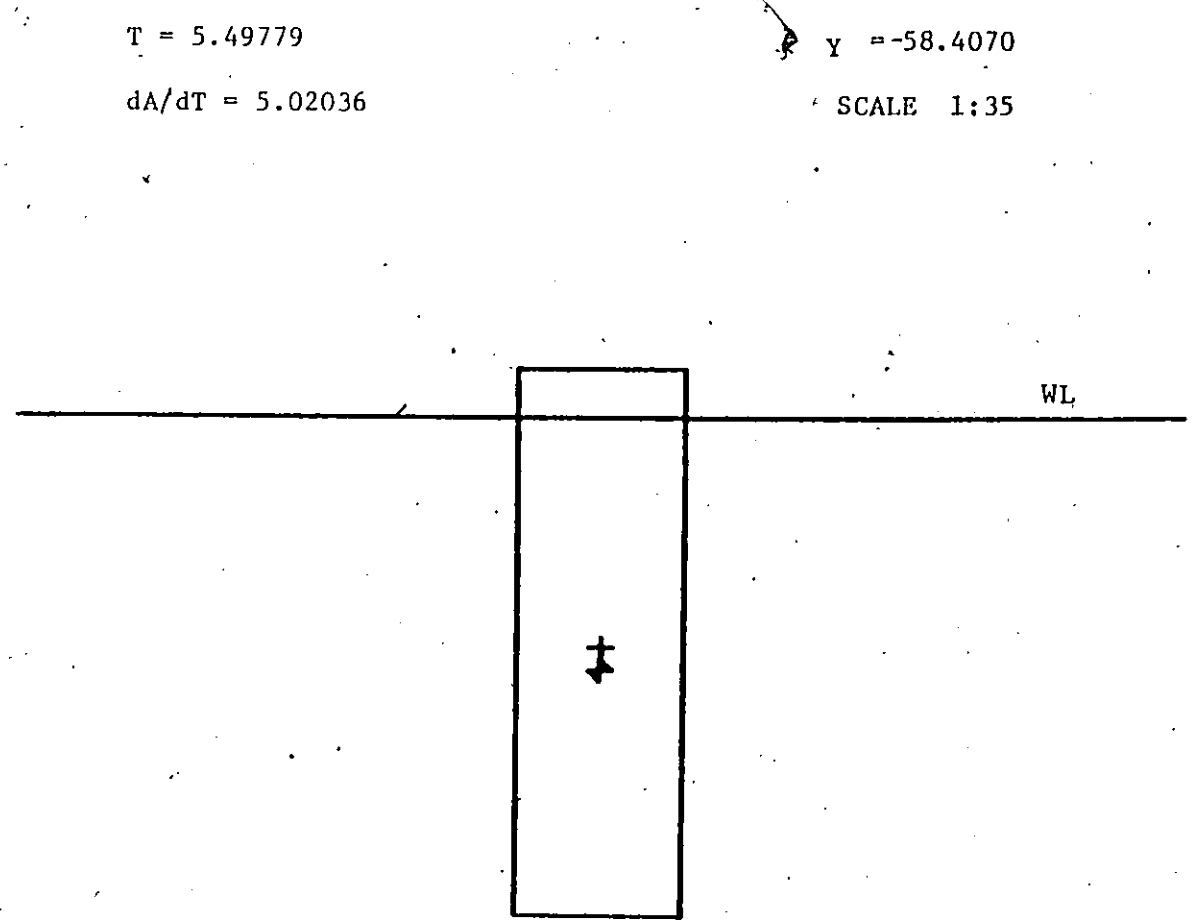


Figure 7d. Unstable Equilibrium Orientation of the Rectangular Shape

Consider two stable orientations one with higher magnitude for dA/dT and another with lower magnitude for dA/dT. For the same change in T, the change in A for the case with higher dA/dT will be higher compared to the change in A for the case with lower dA/dT. Therefore, it can be concluded that the cases with higher magnitudes for dA/dT correspond to more stable orientations than the cases with lower magnitudes for dA/dT. Using this principle, it can be observed that the Figure 5d. corresponds to a more stable orientation compared to the other ones, for that shape. Similarly, for the shape shown in Figure 4b, the orientation shown in Figure 6f is a more stable position compared to the other ones. This algorithm is an exact analysis, because this one is based purely on the fundamental principles.

The following conclusions are drawn from the results obtained in this section.

(1). An algorithm has been developed to find the different orientations of a two dimensional irregular iceberg at which it can float in stable equilibrium on water.

(2). Results obtained from this analysis indicate that a two dimensional iceberg has several stable and unstable equilibrium positions.

(3). From a particular orientation, as the iceberg is rotated through 2π radians, stable and unstable orientations occur one after the other. The number of stable positions must be equal to the number of unstable positions. However, these two conclusions are not true, if an intervening neutral equilibrium position occurs.

(4). Certain stable positions are more stable compared to the other stable positions, for a particular shape.

Thus the procedure described in this section helps in identifying the different stable orientations of a two dimensional irregular iceberg floating on water. This analysis provides a standard against which the results from the numerical scheme used to solve the differential equations can be compared. This also provides the initial orientations for the melting analysis. The validation of the numerical scheme is done in the following section.

3.2 RESULTS OF DYNAMIC ANALYSIS:

In Chapter 2.2, an analysis of forces acting on an iceberg gave rise to a system of equations-one representing the translational motion of the iceberg and the other representing the rotational motion of the iceberg. They were both ordinary differential equations. Solution of those equations represent the motion of the iceberg. A numerical scheme was devised to solve those equations which was self-starting and explicit.

Obviously, there is a necessity to establish that the solution obtained from this numerical scheme represents the true solution. This is done in this section of the report. With the assumption that there is no melting, this algorithm is used in predicting the motion of the iceberg as time progresses from an initial orientation. Naturally, as time progresses, the motion of the iceberg should be such that it brings the iceberg to one of the several stable equilibrium orientations predicted by the stability analysis

For this purpose, a computer program was written. This program will predict the motion of the iceberg from an initial orientation, for a maximum specified time. This is given in Appendix C.

The motion of each of the three icebergs considered in this work was predicted for three initial orientations(which correspond to three initial conditions). In all the cases the Δt chosen was 0.2 seconds. The total time of the analysis was chosen to be 300 seconds. The translational displacement versus time as well as the angular displacement versus time was plotted for these initial conditions.

The Figures 8a and 8b show the result of this type of analysis applied to the typical iceberg shown in Figure 4a. The initial orientation chosen correspond to Y=-3.0 m and T=0.1 rad. These graphs indicate that at the end of 300 seconds, the iceberg

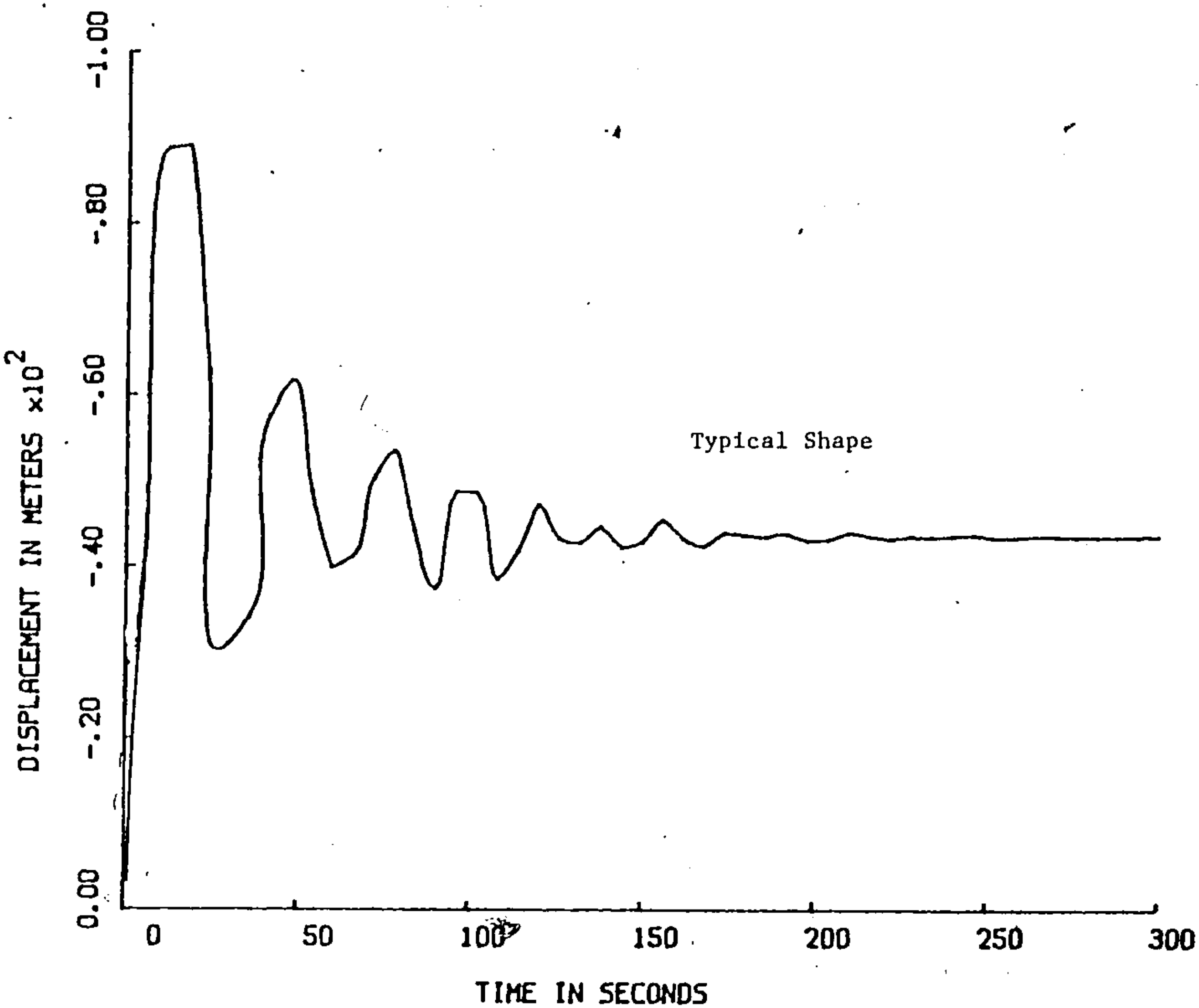


Figure 8a. Variation of Translational Displacement with Time

(Y_{INI} = -3.0 m and T_{INI} = 0.1 rad)

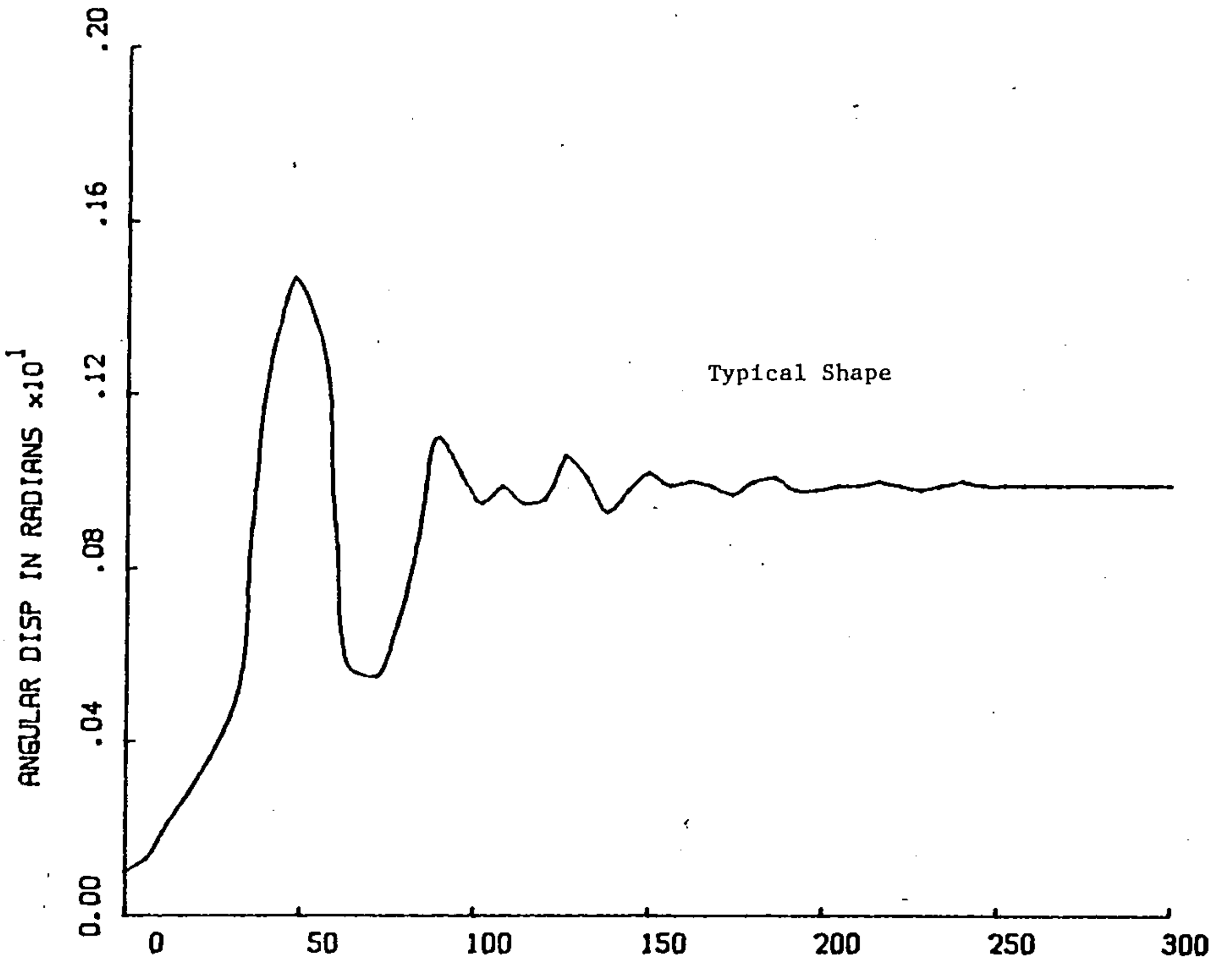


Figure 8b. Variation of Angular Displacement with Time (Y_{INI}=-3.0 m and T_{INI}=0.1 rad)

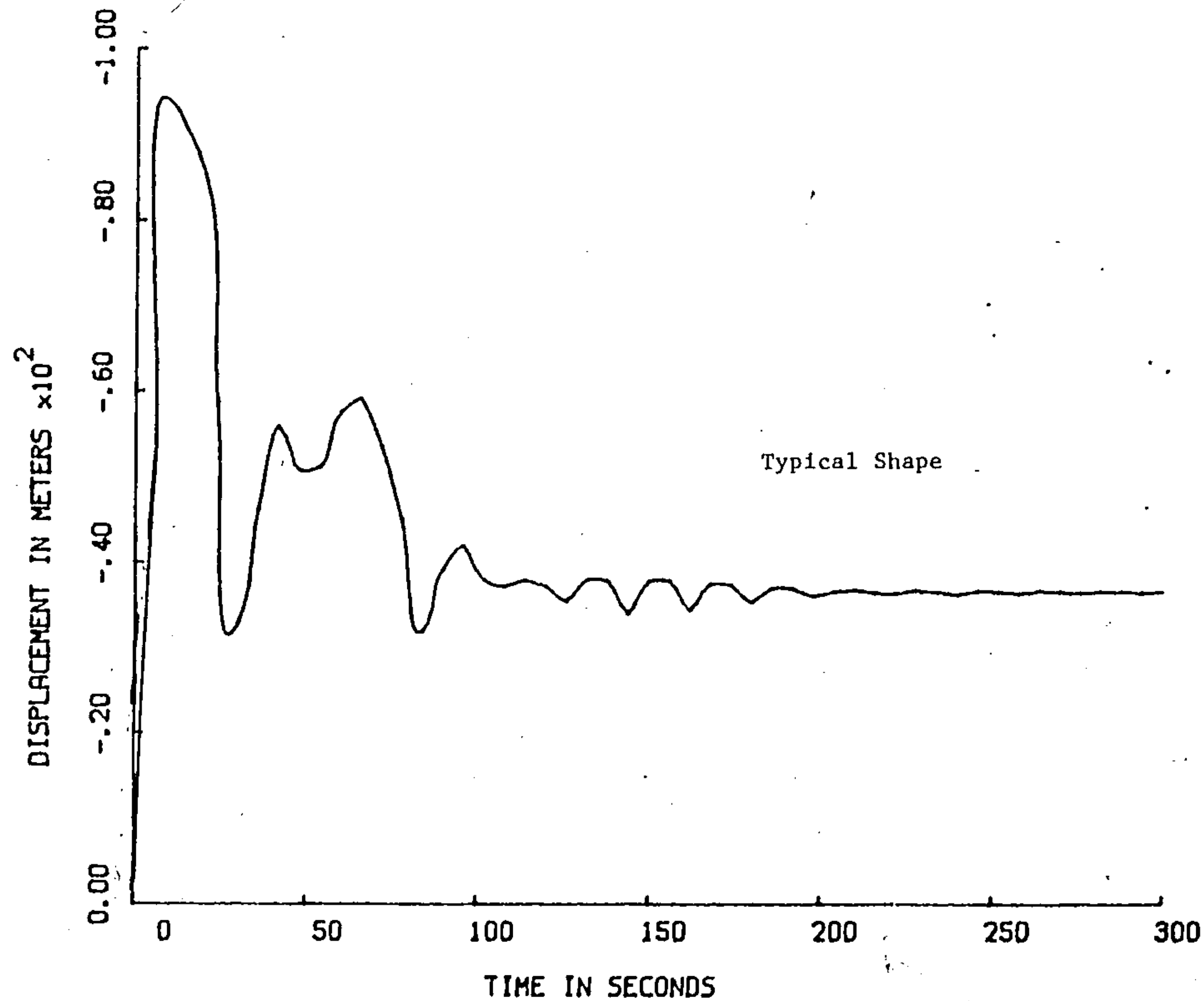


Figure 8c. Variation of Translational Displacement with Time (Y_{INI} = -3.0 m T_{INI} = 1.5 rad)

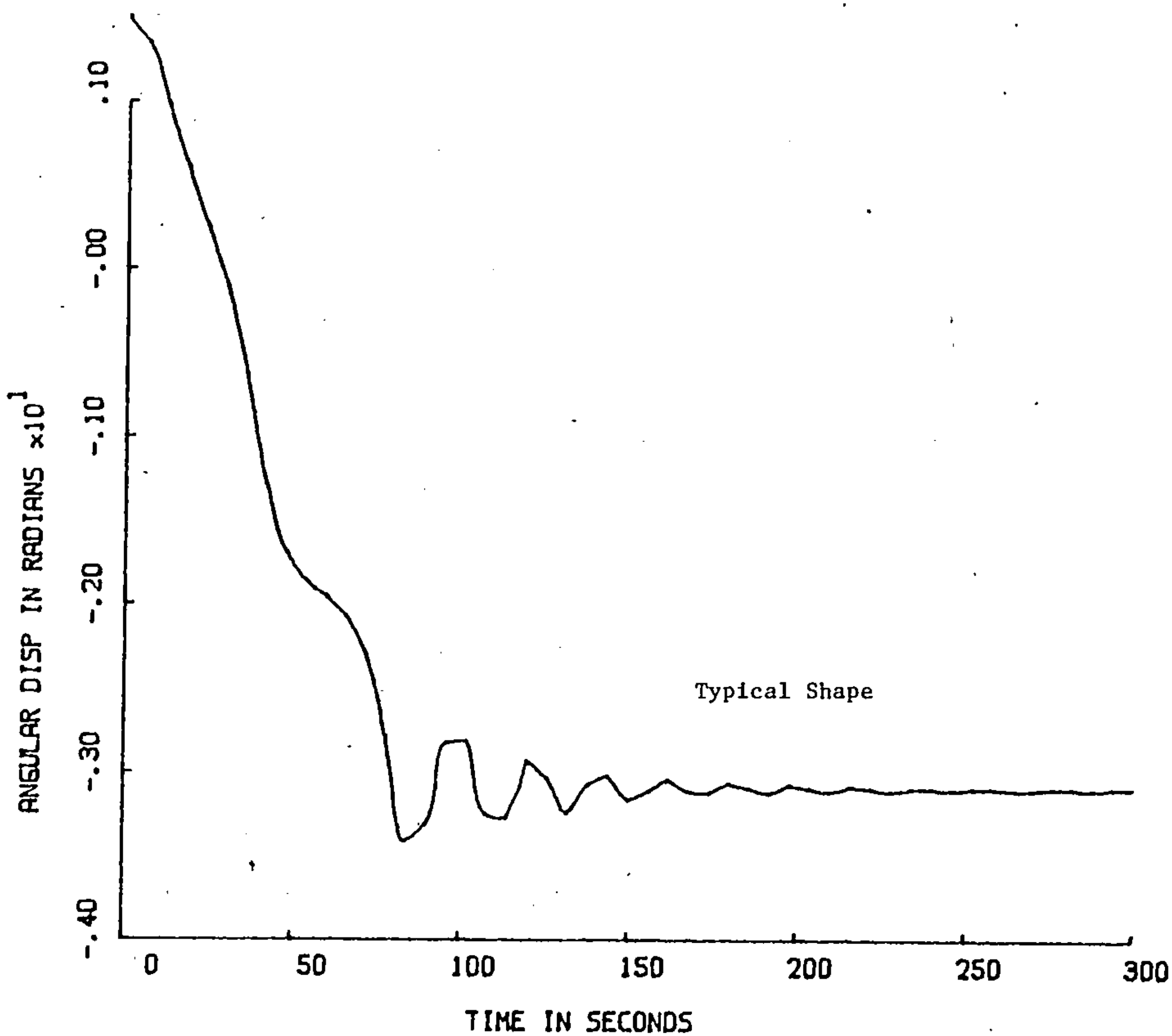


Figure 8d. Variation of Angular Displacement with Time (Y_{INI} = -3.0 m and T_{INI} = 1.5 rad)

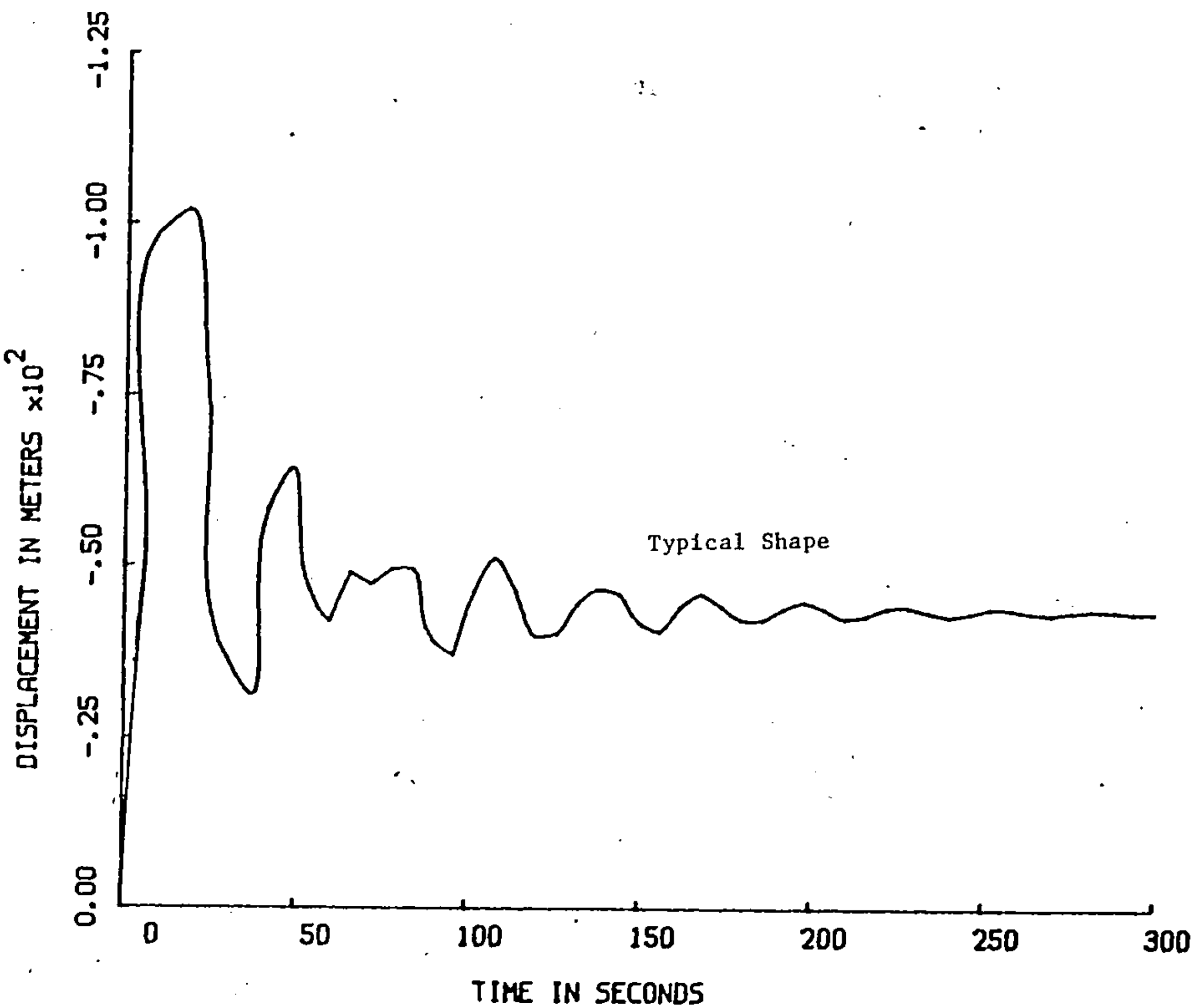


Figure 8e. Variation of Translational Displacement with Time (Y_{INI} = -3.0 m and T_{INI} = 4.5 rad)

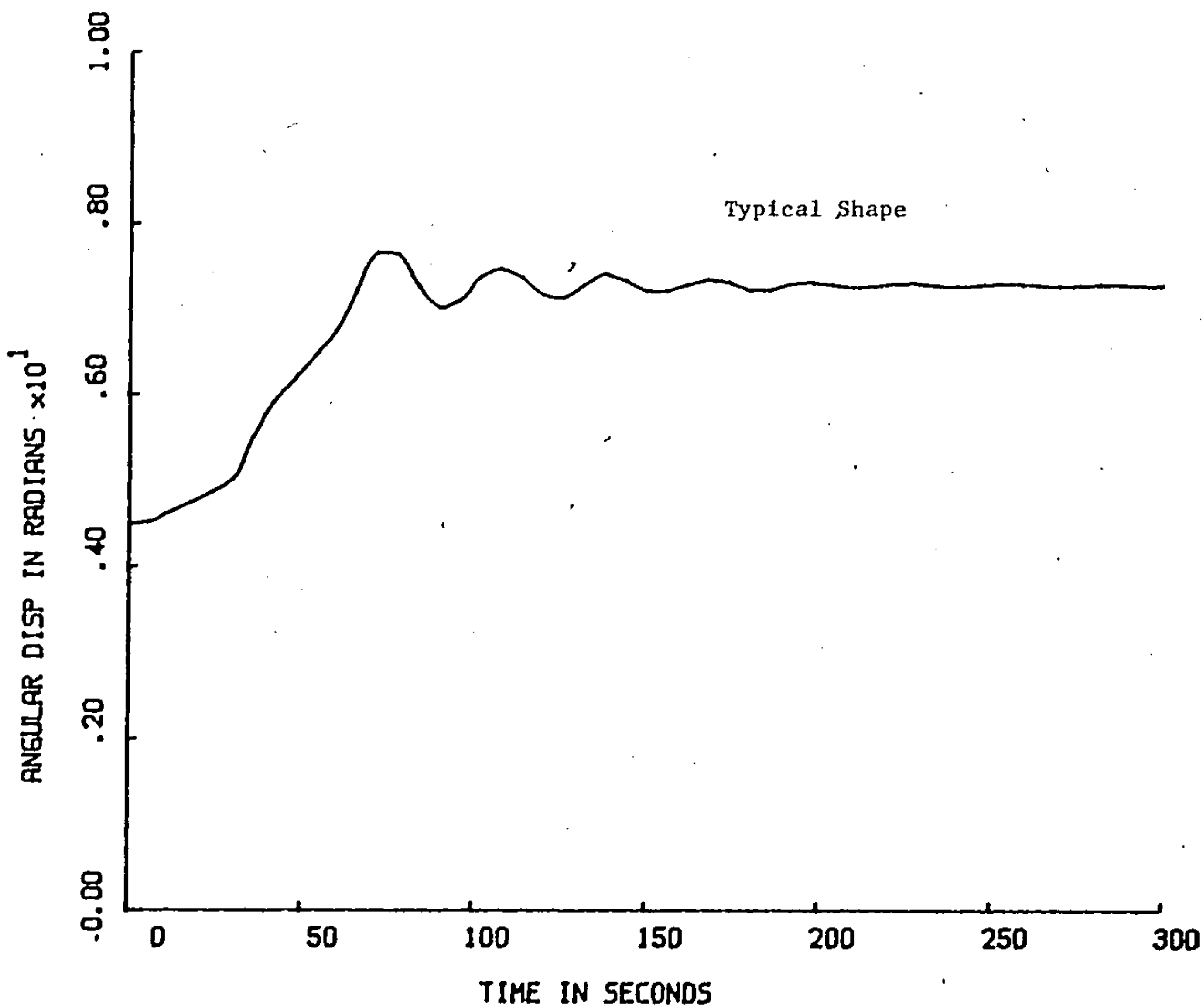


Figure 8f. Variation of Angular Displacement with Time (Y_{INI} = -3.0 m and T_{INI} = 4.5 rad)

comes to a position identified by Y=-43.6512 m and T=0.9928 rad. This orientation corresponds very closely to the orientation shown in Figure 5b. The orientation shown in Figure 5b is one of the stable orientations for the typical shape and is identified by Y=-43.5869 m and T=0.9923 rad. The Y value predicted by the numerical scheme is higher by 0.0643 m compared to the Y value corresponding to Figure 5b. The agreement between both the T values is quite good, for the difference between them is only 0.0005 radians.

Figures 8c and 8d show the results of the dynamic analysis for the typical shape when the initial orientation is given by Y=-3.0 m and T=1.5 rad. Figure 8c and Figure 8d indicate that at the end of 300 seconds, the value of Y is -36.5044 m and the value of T is -3.093 rad. A T value of -3.093 rad corresponds to the same orientation identified by T=3.19018 rad. These Y and T values correspond very closely to the Y and T values corresponding to the orientation shown in Figure 5d. The Y and T values corresponding to Figure 5d are -36.5131 m and T=3.18985 rad. Obviously, the results predicted by the numerical scheme agree very closely with the values corresponding to the stable orientation.

Figures 8e and 8f show the result of the dynamic analysis for the typical shape, when the initial orientation is identified by Y=-3.0 m and T=4.5 rad. At the end of 300 seconds, the numerical solution brings the iceberg to a position identified by Y=-43.3916 m and T=7.2702 rad. The angular orientation given by T=7.2702 rad is the same as the angular orientation given by T=0.9870 rad. This orientation is approximately the same as the orientation shown in Figure 5b. The difference between the Y values in this case is 0.1953 m. This difference is higher compared to the previous two cases. The difference between the T values is 0.0053 rad.

In order to show that the procedure developed in this work can be applied to an irregular shape with a hole in it, the same algorithm is applied to the shape shown in Figure 4b. In this case also the initial conditions are taken to be the same as that for the previous shape. The results of this analysis are shown in Figures 9a to 9f.

When the initial conditions are Y=-3.0 m and T=0.1 rad, the results are shown in Figure 9a and 9b. From these Figures, it is clear that the iceberg comes to an orientation identified by Y=-57.2081 m and T=-0.1036 rad, at the end of 300 seconds. These Y and T values correspond very closely to the Y and T values for the orientation shown in Figure 6f. The orientation in Figure 6f is one of the stable positions for that shape. The value of Y predicted by the numerical scheme at the end of 300

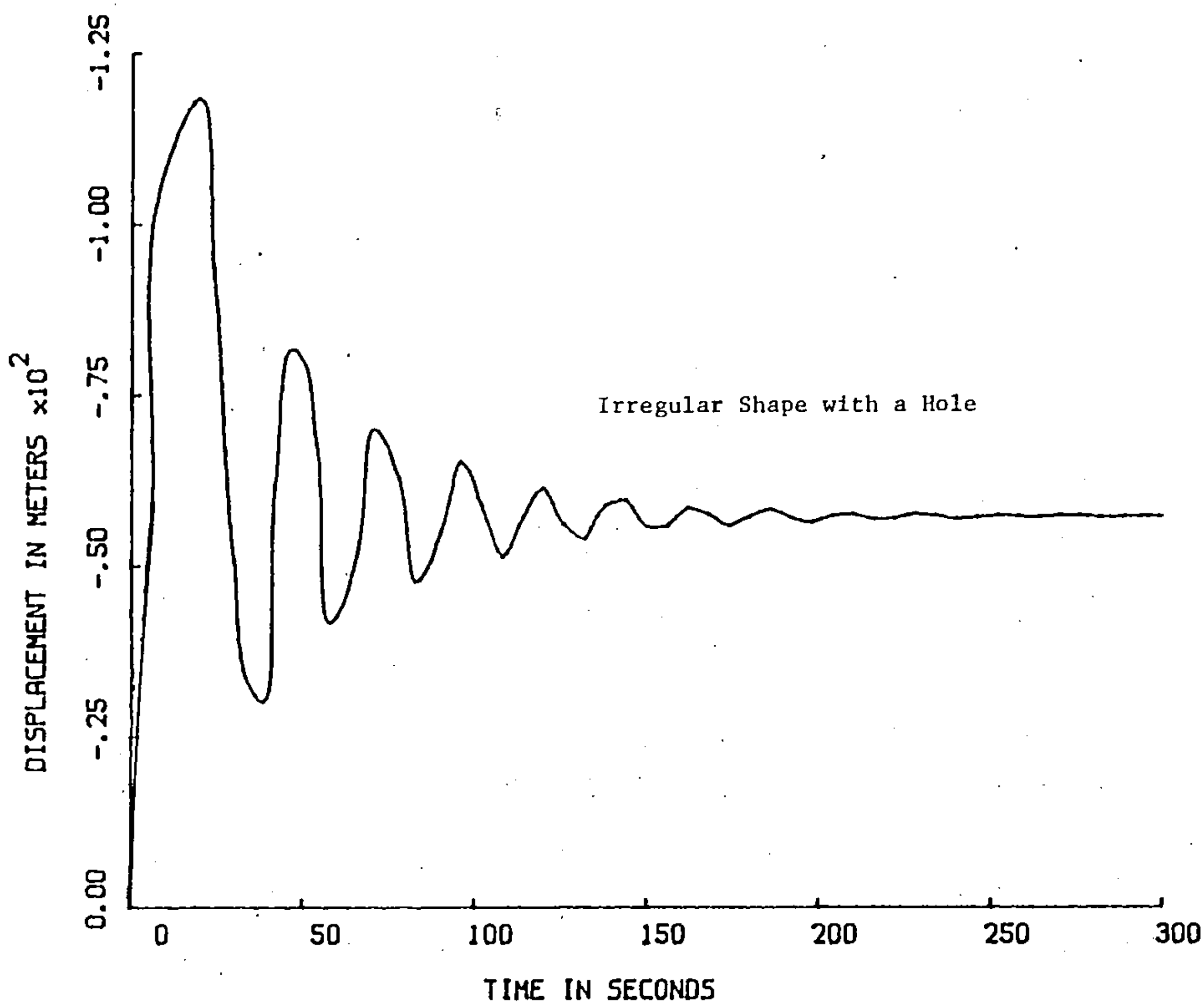


Figure 9a. Variation of Translational Displacement with Time ($Y_{INI} = -3.0$ m and $T_{INI} = 0.1$ rad)

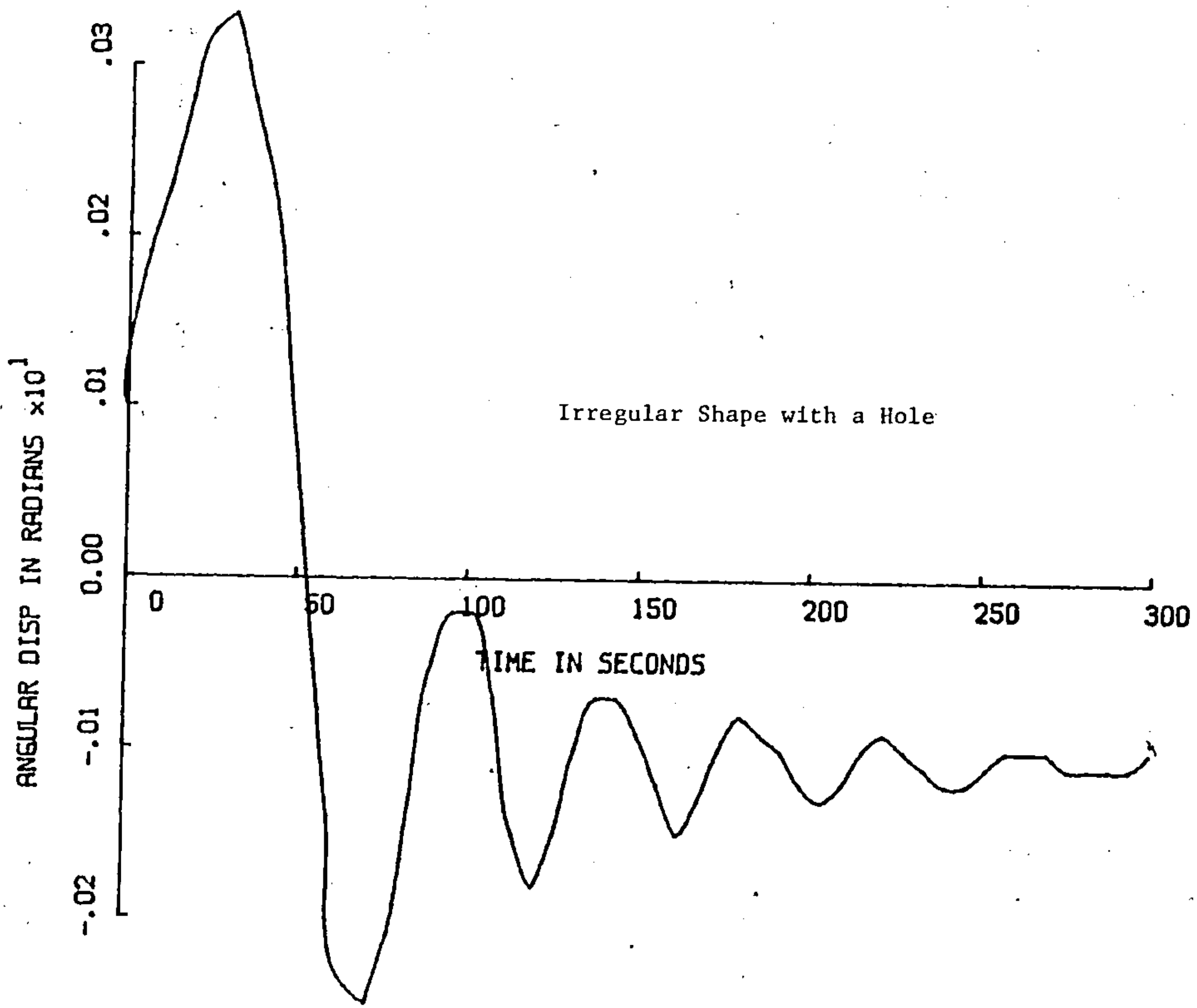


Figure 9b. Variation of Angular Displacement with Time (Y_{INI} = -3.0 m and T_{INI} = 0.1 rad)

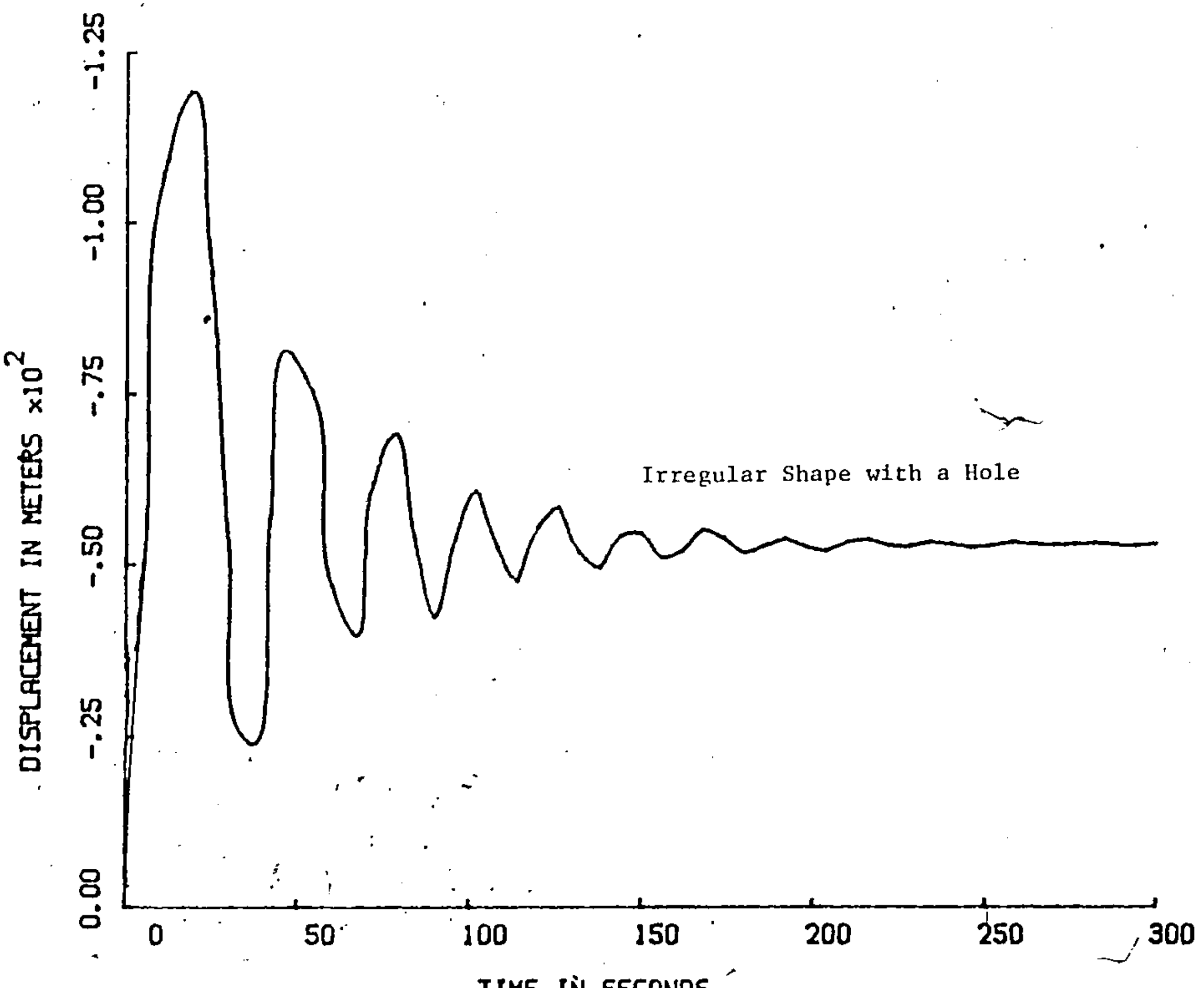


Figure 9c. Variation of Translational Displacement with Time (Y_{INI} = -3.0 m and T_{INI} = 1.5 rad)

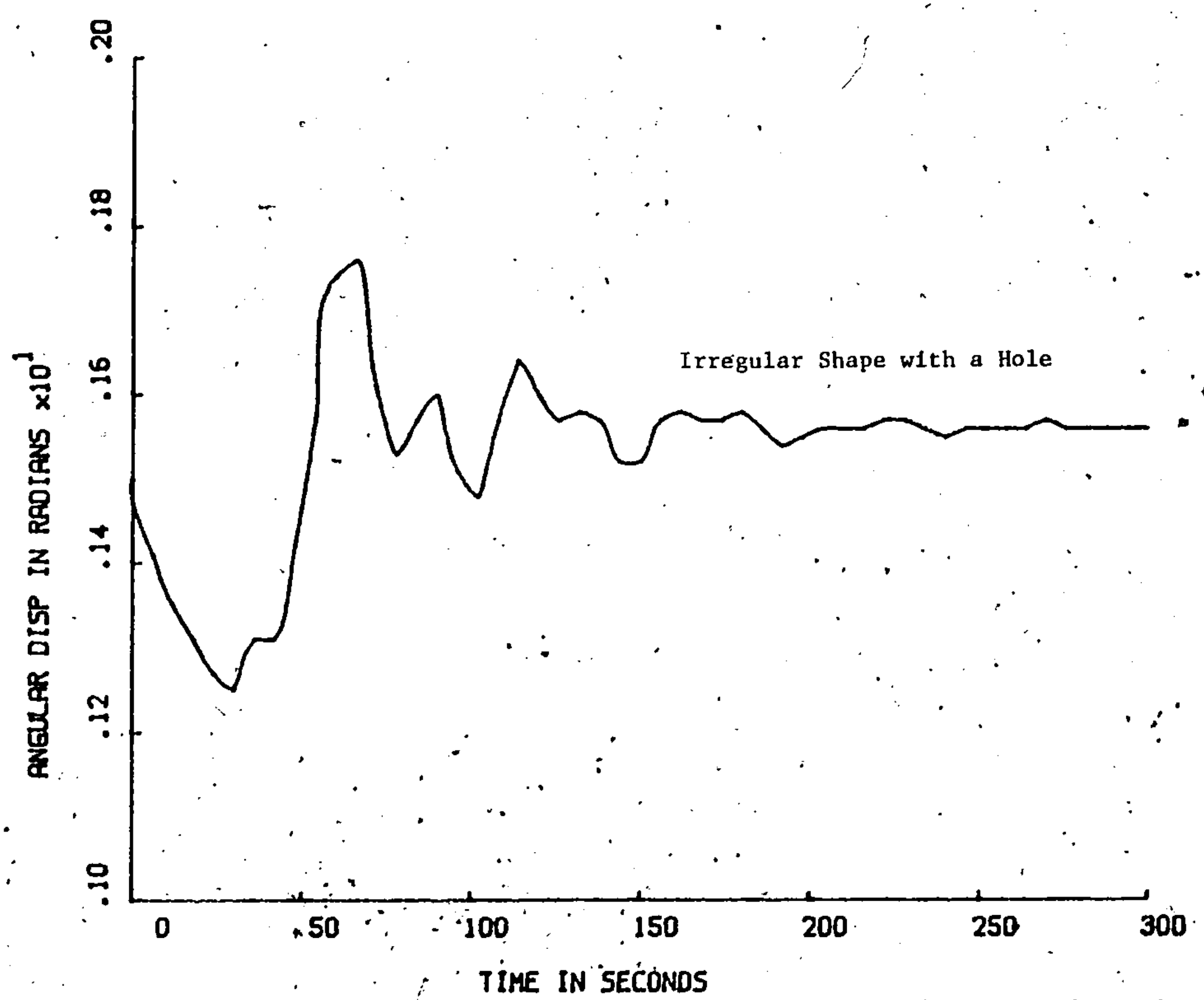


Figure 9d. Variation of Angular Displacement with Time (Y_{INI} = −3.0 m and T_{INI} = 1.5 rad)

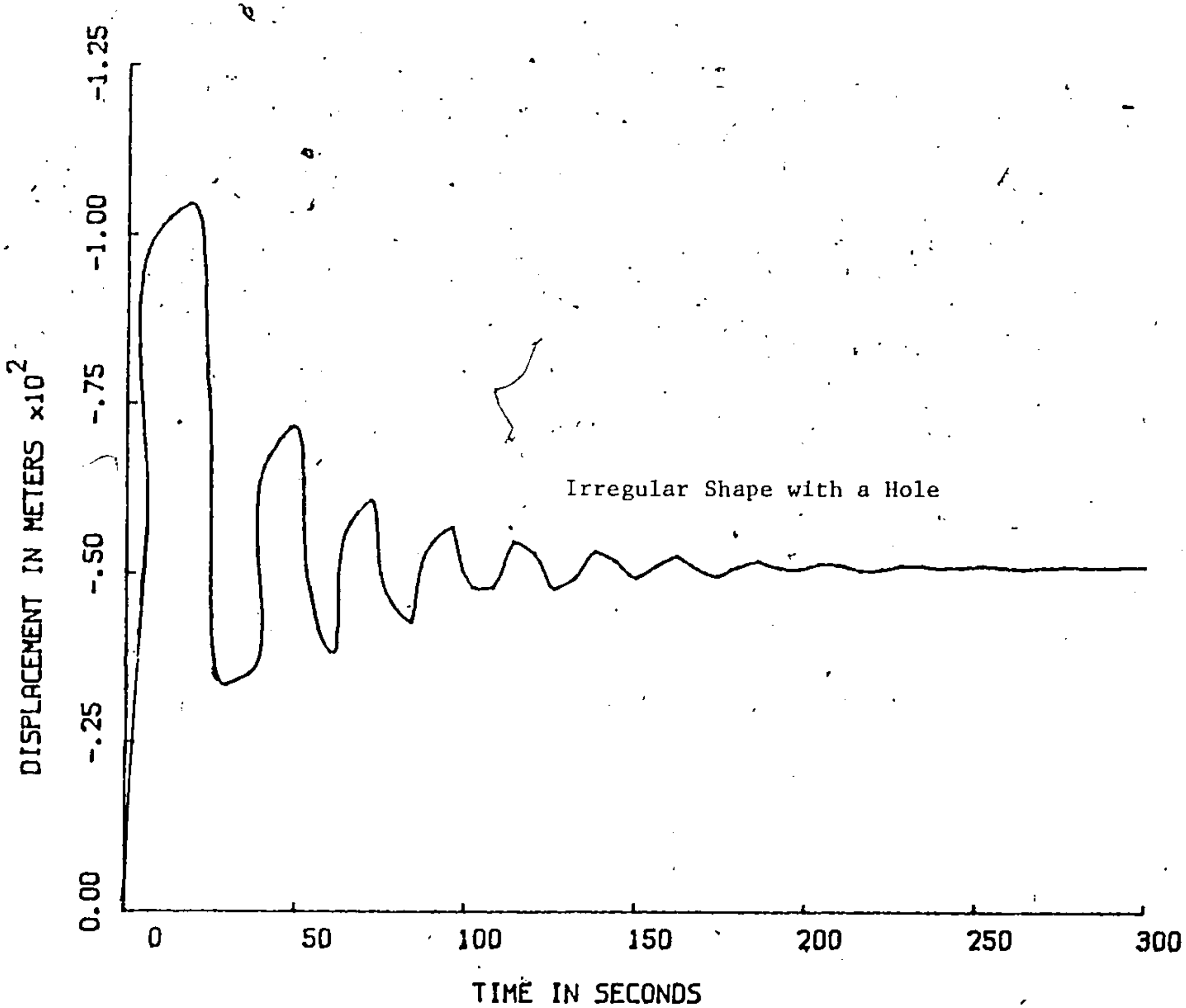


Figure 9e. Variation of Translational Displacement with Time (Y_{INI} = -3.0 m and T_{INI} = 4.5 rad)

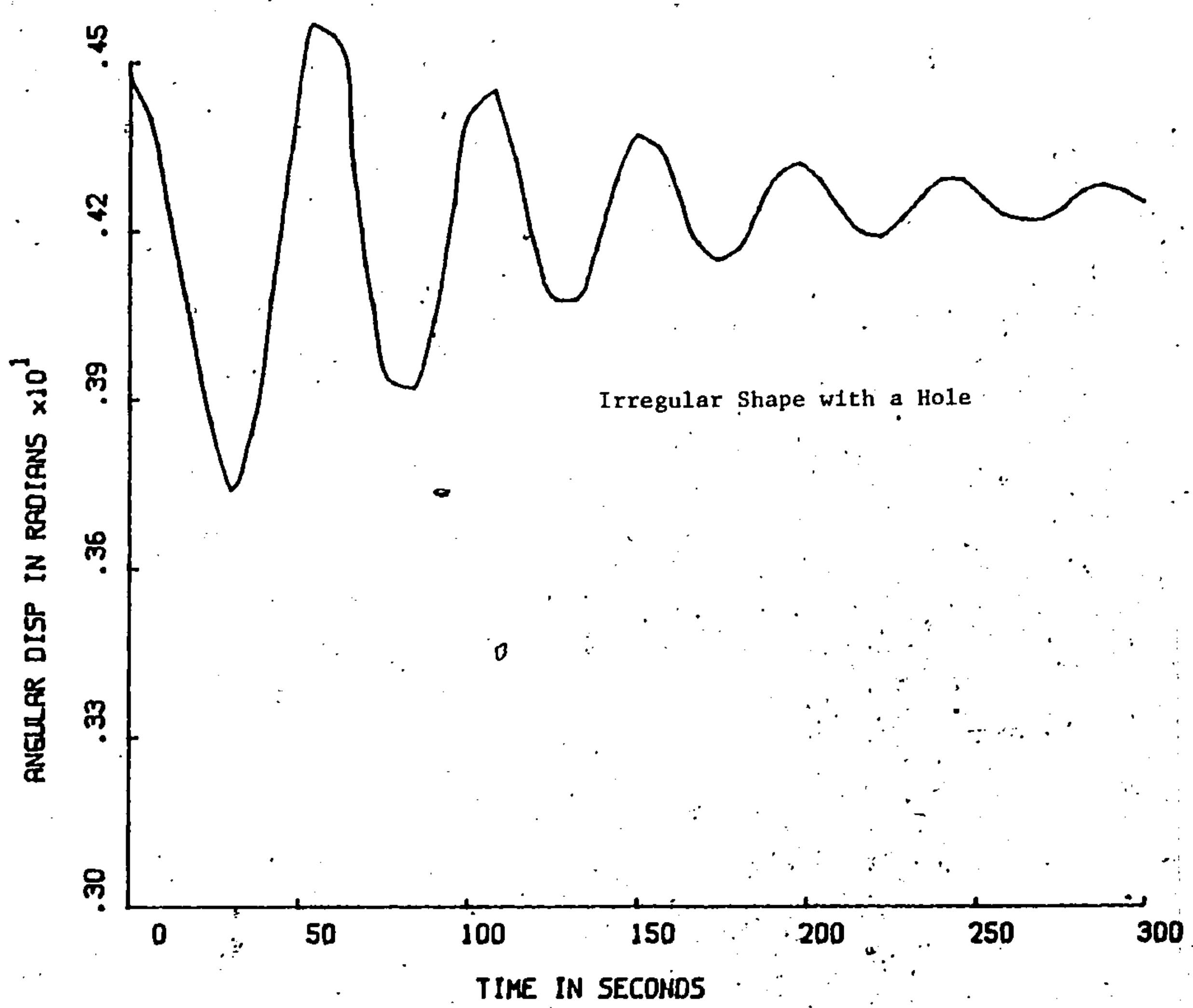


Figure 9f. Variation of Angular Displacement with Time (Y_{INI} = -3.0 m and T_{INI} = 4.5 rad)

seconds is lower by 0.043 m compared to Figure 6f. A T value of -0.1036 rad is the same as a T value of 6.1796 rad. The T value predicted by the numerical solution is higher by 0.0033 m compared to the T value corresponding to the Figure 6f. However, an observation of Figure 9b indicates that the oscillation tendency of the angular displacement has not died out completely even at 300 seconds. At the same time, the curve does not indicate any tendency of deviation from the position about which it is oscillating and also the amplitude of oscillations is decreasing. The graph shows that the angular displacement starts oscillating in a decaying fashion from around 100 seconds. The graph reaches a local minimum at about 120 seconds and has an angular displacement of -0.1815 rad. At 286 seconds, the graph reaches another local minimum which corresponds to an angular displacement of -0.1132 rad. The T value corresponding to the orientation shown in Figure 6f is -0.1068 rad. The amplitude of the angular displacement about this T value at 120 seconds is 0.0747 rad and at 286 seconds is 0.0064 rad. The ratio of these amplitudes is only 0.086. This means that the amplitude at 286 seconds is only 8.6 % of the amplitude at 120 seconds. Therefore, even though the curve shows an oscillating behaviour, the amplitude of oscillations is greatly reduced at about 300 seconds. Therefore, the T value at 300 seconds is taken to represent the final angular orientation of the iceberg.

Figures 9c and 9d show the results of the dynamic analysis for the irregular shape with a hole, when the initial orientation is identified by Y=-3.0 m and T=1.5 rad. At the end of 300 seconds, the Y and T values are -53.0346 m and 1.559 rad respectively. These Y and T values correspond approximately to that of the orientation shown in Figure 6b. The orientation shown in Figure 6b is one of the stable positions for that shape. The Y and T values corresponding to Figure 6b are -53.0154 m and 1.56121 rad respectively. Obviously, the agreement between the final orientation predicted by the numerical scheme with the stable position shown in Figure 6b is quite good.

Finally, Figures 9e and 9f give the result of the dynamic analysis, when the initial orientation is given by Y=-3.0 m and T=4.5 rad. The orientation at the end of 300 seconds is identified by Y=-51.141 m and T=4.2492 rad. These Y and T values correspond approximately to the orientation shown in Figure 6d. The Y value corresponding to Figure 6d is -51.0933 m and the T value is 4.25037 rad. Obviously, the agreement is quite good. However, the angular displacement versus time curve shown in Figure 9f shows an oscillating behaviour even at 300 seconds. The graph starts oscillating in a decaying fashion from about 50 seconds. The curve reaches a local maximum at 56.6 seconds and the angular orientation is 4.5976 rad. It has another local maximum at 293.8 seconds and the angular orientation is 4.2677 rad. If the curve is assumed to oscillate about the T value corresponding to the orientation shown in Figure 6d, then obviously the ratio of the amplitude of oscillation about this T value gives a measure of how much of the oscillation has died out. This ratio in this case is 0.0053. Therefore, even though the graph shows an oscillating behaviour, the amplitude of oscillation is reduced drastically at 300 seconds. Therefore, the angular orientation given by this graph at 300 seconds is taken to be the final angular orientation predicted by the numerical scheme.

Finally, the same algorithm was applied to the rectangular shape shown in Figure 4c. Figures 10a and 10b show the result of this analysis, when the initial condition is given by Y=-3.0 m and T=-0.786 rad. Figures 10a gives the displacement versus time curve, while Figure 10b gives the angular displacement versus time. The displacement curve reaches a constant value at about 150 seconds. The value of Y at the end of 300 seconds is -17.54 m. The angular displacement curve reaches a steady state value at about 200 seconds. The value of T at the end of 300 seconds is 0.7859 rad. These Y and T values correspond very closely to the Y and T values in Figure 7a. The Figure 7a is one of the stable positions for the rectangular shape. The Y and T values corresponding to the Figure 7a are -17.5221 m and 0.785397 rad respectively. The value of Y

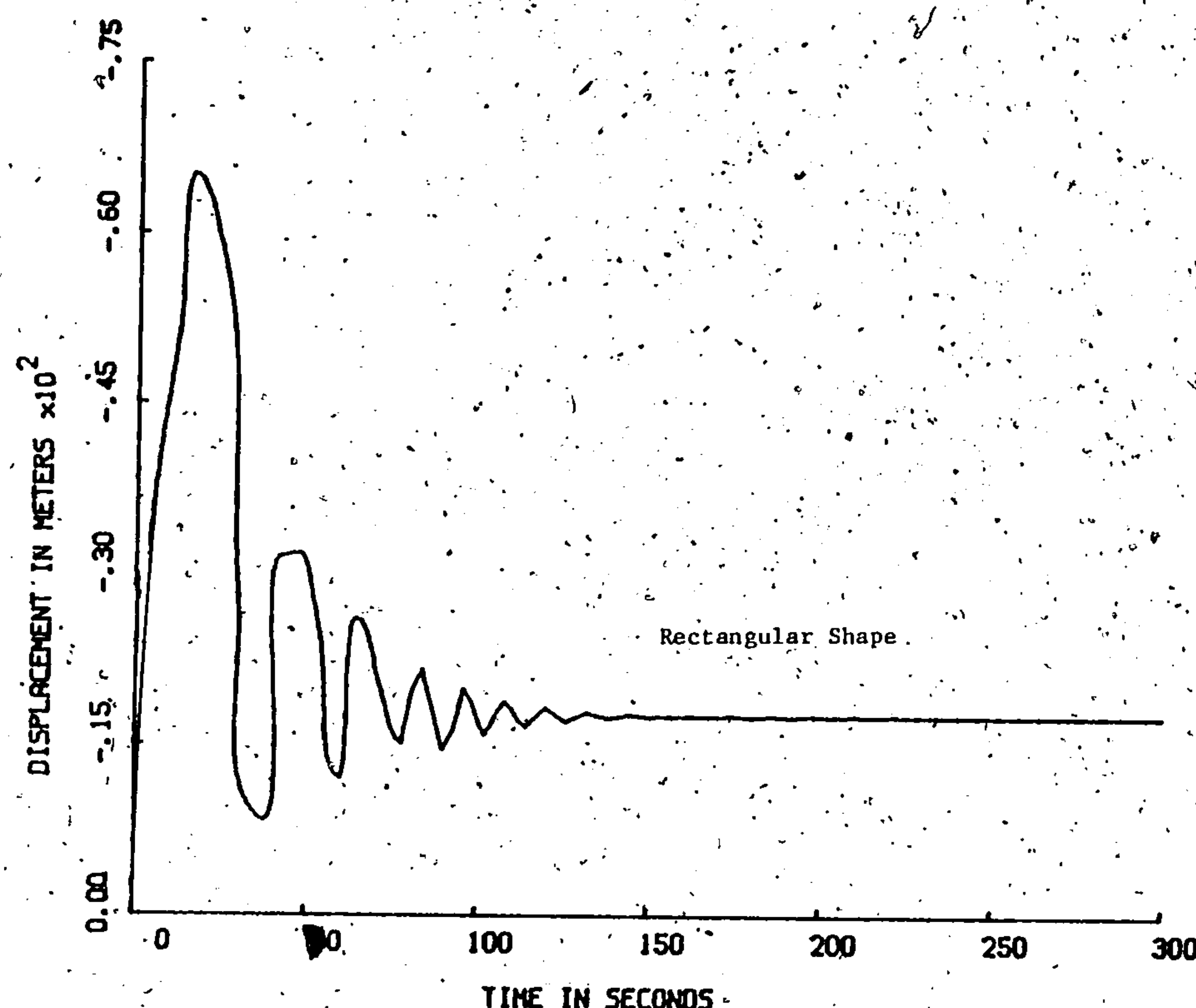


Figure 10a. Variation of Translational Displacement with Time (Y_{INI} = -3.0 m and T_{INI} = -0.786 rad)

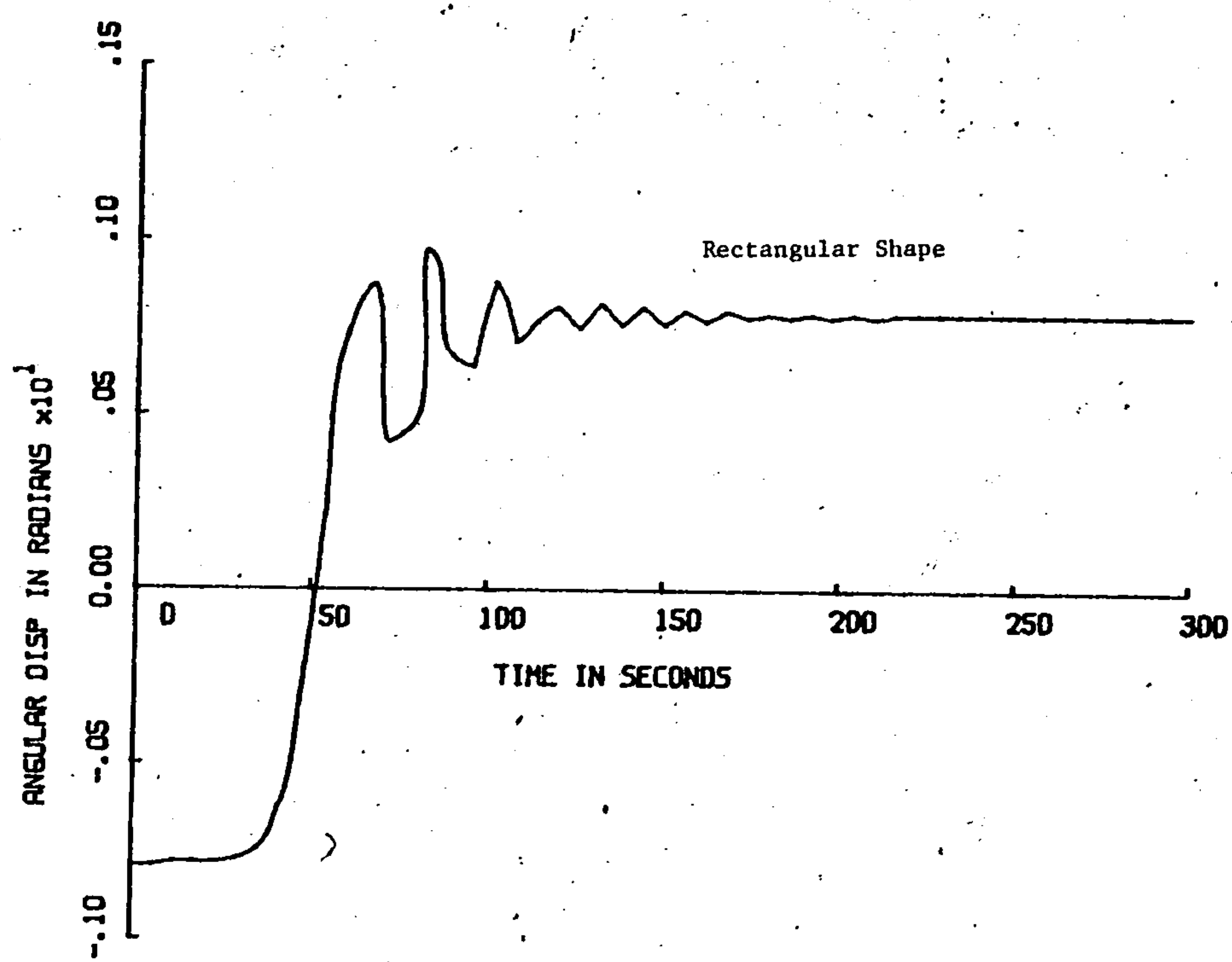


Figure 10b. Variation of Angular Displacement with Time (Y_{INI} = -3.0 m and T_{INI} = -0.786 rad)

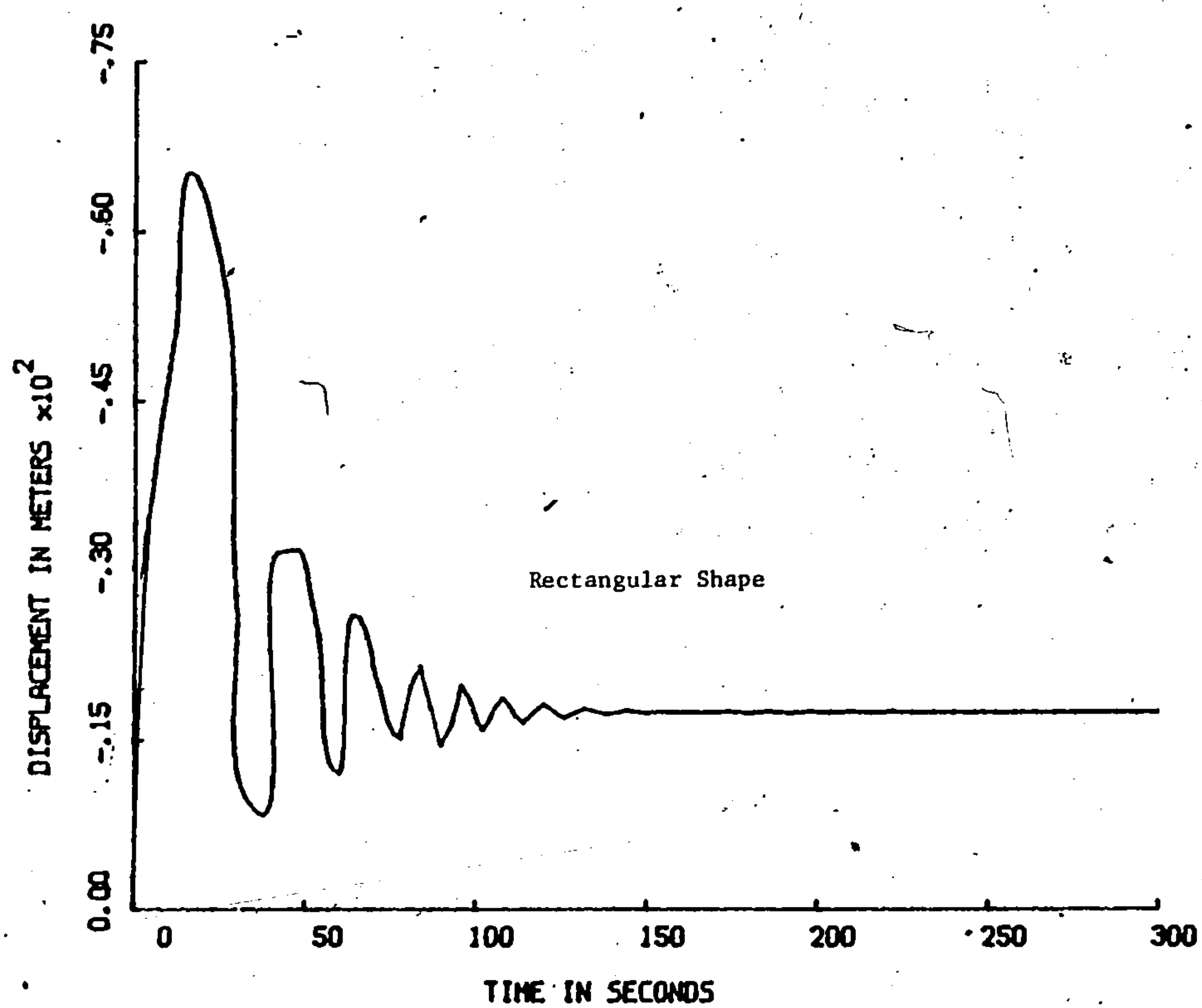


Figure 10c. Variation of Translational Displacement with Time (Y_{INI} = -3.0 m and T_{INI} = 1.5 rad)

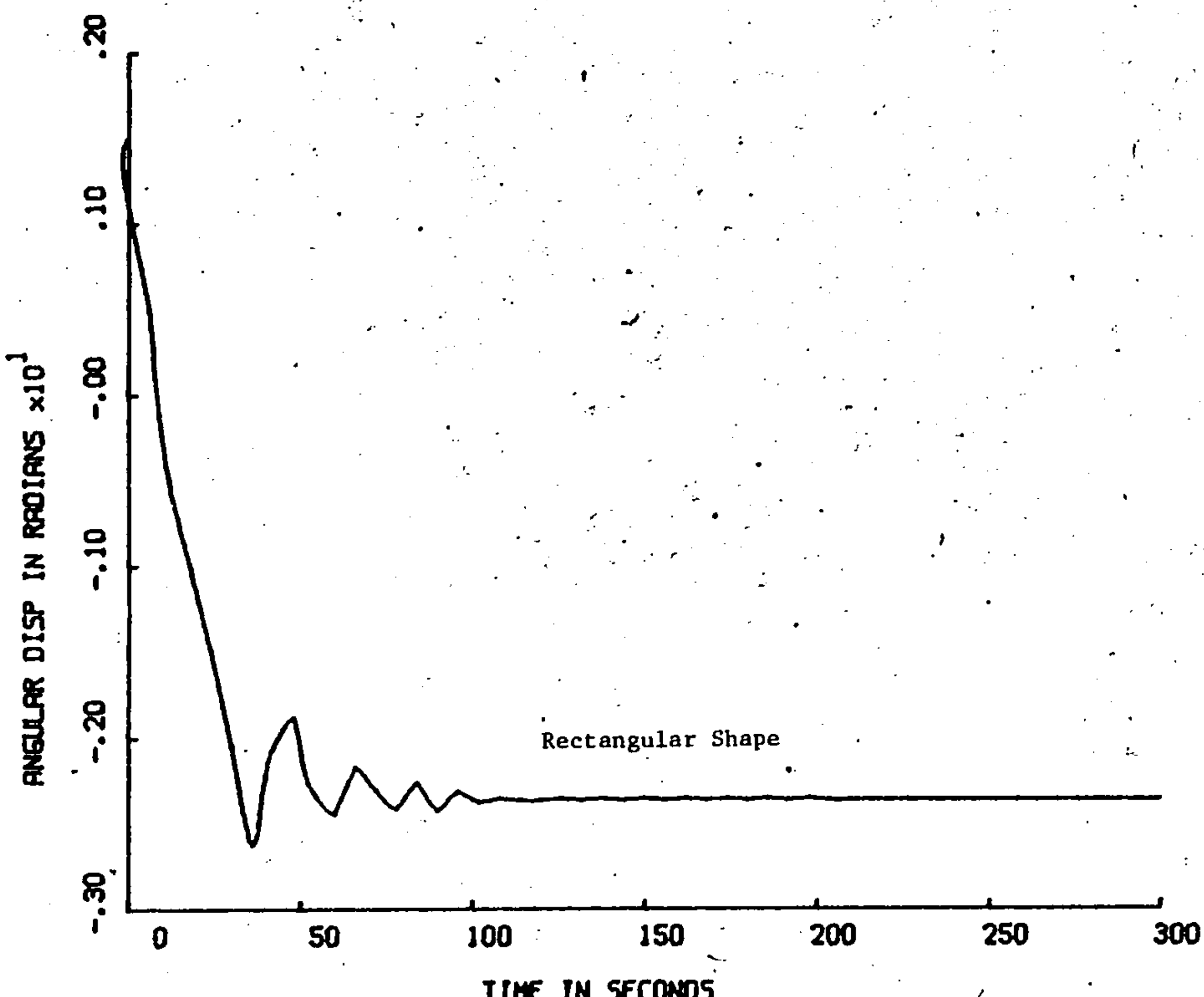


Figure 10d. Variation of Angular Displacement with Time (Y_{INI} = -3.0 m and T_{INI} = 1.5 rad)

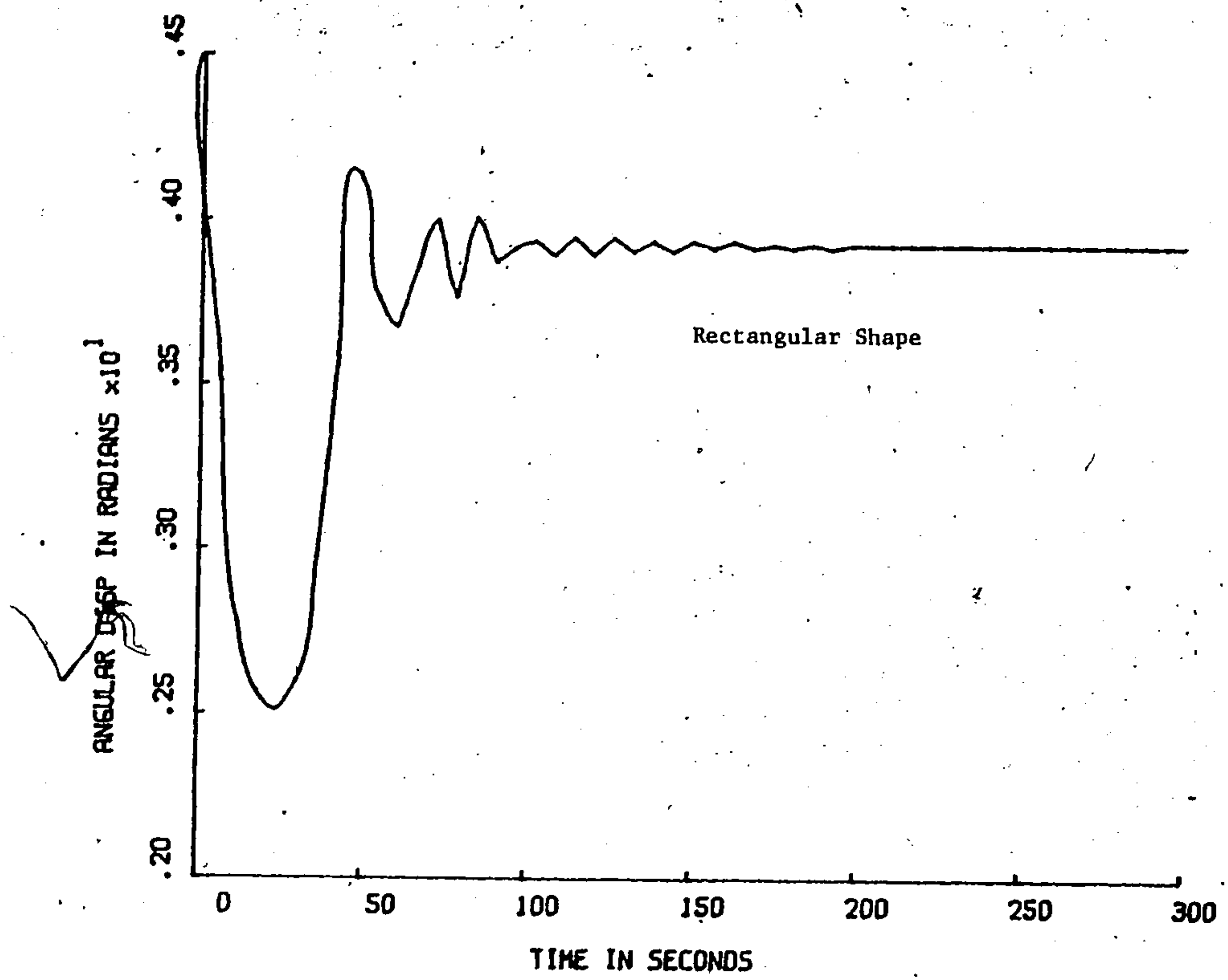


Figure 10f. Variation of Angular Displacement with Time (Y_{INI} = −3.0 m and T_{INI} = 4.5 rad)

predicted by the numerical method is lower by 0.02 m compared to the Y corresponding to Figure 7a. The value of T is higher by 0.0006 rad compared to the T for Figure 7a.

Figures 10c and 10d show the results of the dynamic analysis applied to the rectangular shape when the initial condition is given by Y=-3.0 m and T=1.5 rad. The orientation at the end of 300 seconds is given by Y=-17.52 m and T=-2.45 rad. T=-2.45 rad is the same as T=3.833 rad. These Y and T values correspond very closely to the same orientation shown in Figure 7a.

Figures 10e and 10f show the results of the dynamic analysis applied to the rectangular shape, when the initial condition is given by Y=-3.0 m and T=4.5 rad. As shown by these Figures, the oscillating behaviour of the curve dies out completely at about 150 seconds. At the end of 300 seconds, the Y and T values are -17.52 m and 3.95 rad respectively. These Y and T correspond approximately to the Y and T values for the orientation shown in Figure 7c. The Figure 7c is one of stable orientations for the rectangular shape. Table 1 gives the final orientation corresponding to each initial orientation for the three shapes.

The next important thing that is to be discussed is the stability of the numerical method. This numerical scheme is not

TABLE 1

RESULTS OF DYNAMIC ANALYSIS

SHAPE	INITIAL ORIENTATION		FINAL ORIENTATION		
	Y	T	Y	T	
Fig. 4a	-3.0	0.1	-43.6512	0.9928	Fig. 5b
	-3.0	1.5	-36.5044	-3.093	Fig. 5d
	-3.0	4.5	-43.3916	7.2702	Fig. 5b
Fig. 4b	-3.0	0.1	-57.2081	-0.1036	Fig. 6f
	-3.0	1.5	-53.0365	1.559	Fig. 6b
	-3.0	4.5	-51.141	4.2492	Fig. 6d
Fig. 4c	-3.0	-0.78539	-17.52	0.7859	Fig. 7a
	-3.0	1.5	-17.52	-2.45	Fig. 7c
	-3.0	4.5	-17.52	3.95	Fig. 7c

stable for all values of Δt and as a matter of fact, it has different stability limits for different shapes. For example, the method is stable for the typical shape shown in Figure 4a, for Δt values less than or equal to 5 seconds. For the irregular shape with a hole shown in Figure 4b, the method is stable for Δt values less than or equal to 6 seconds. For the rectangular shape, it is stable for Δt values between 0 and 8 seconds.

The reasons for the lower values of Δt for the method to be stable are several.

(1). Since the shapes considered are quite irregular, a small change in T or Y or in both can bring a substantial change in B. This really means that the B is very sensitive to a small change in Y or T or in both. Since the dependency of B on Y and T is different for different shapes, obviously, the method has different stability limits for different shapes.

(2). Another reason for the lower values of Δt , is that the finite difference schemes used in equations 7a and 8a are explicit.

Now, two important questions arise - firstly, are lower values of Δt acceptable ? - secondly, since the method has different stability limits for different shapes, how does one find out the stability limits for different shapes ? The following paragraphs answer these questions.

Lower values of Δt should be chosen for two reasons – firstly because lower values of Δt improve accuracy and secondly because of limits imposed by the stability. Whether lower values of Δt are agreeable or not really depends on the computer execution time. For 300 seconds of modelled time with Δt =0.2 seconds, the program takes 1.5 minutes of CPU time. This execution time is obviously not very high and thus, a value of Δt =0.2 seconds is acceptable.

The following method is suggested to find out the stability limits for different shapes. It is necessary to run some trial runs to find out the stability limit. This can be done by running the computer program for a particular Δt from 0 to 20 seconds of motion modelling time. If the method is unstable for the Δt chosen, then the following things may happen:

1. The displacement between time intervals may become so high that at a particular time, the iceberg may be above the waterline.

2. During the next time interval, the entire iceberg may be well below the waterline surface.

3. Similar behaviour with the angular displacement may also be noticed.

In this way, by running some trial runs, the stability limit can be established.

From the results obtained in this section of the report, the following conclusions are drawn.

(1). No matter what the initial condition is, the iceberg comes to one of the several stable positions predicted by the previous algorithm. The difference between the values of Y ranges from 0.1953 m to 0.0021 m. The difference between the values of T ranges from 0.117 rad to 0.0005 rad. Even though oscillations are observed in some angular displacement versus time curves, it is found that the amplitude of oscillations is greatly reduced at 300 seconds.

(2). The final orientation of the iceberg at the end of 300 seconds is dependent on the initial condition.

(3). From the graphs shown in Figures 9a to 11f, it is clear that within 300 seconds, the iceberg comes to one of the stable positions. This really suggests that 300 seconds of modelling time is quite sufficient for stability to be achieved.

3.3 RESULTS OF MELTING ANALYSIS:

In Chapter 2.3, a procedure was devised to model the motion of the iceberg arising because of its shape changing due to melting. This procedure required that the different stable positions of a two dimensional irregular iceberg be known. This was done in Chapter 3.1, where a procedure was described to find the different stable positions of a two dimensional irregular iceberg. By this procedure, it was established that a two dimensional iceberg has several stable positions.

The melting analysis also required that the numerical scheme used to solve the differential equations be validated. This validation was done in Chapter 3.2. The results from this validation study indicate that 300 seconds of motion modelling time is quite sufficient for stability to be achieved.

In order to model the roll over phenomenon, it is also necessary that the melting analysis should be carried out in the following manner.

(1). The initial orientation should be chosen to be one of the stable positions. The stable orientations for the icebergs considered in this work are given by the results of the stability analysis.

(2). Starting from the stable position and using the melting model, the change in shape of the iceberg should be predicted for each time step Δt. The motion of the iceberg should be modelled using the dynamic analysis. This procedure should be continued till the time roll-over occurs. Roll-over is a major change in the angular orientation of the iceberg caused by melting. Quantitatively, if an angular displacement of 1 radian is caused by melting, then it is taken that the iceberg has rolled over.

The roll-over time is usually of the order of days. However, the time step Δt is restricted to within seconds, by the stability limit imposed by the numerical method. The execution time of the computer program will be tremendously high, if the time of melting is taken to be within the time step Δt, imposed by the stability limit. This problem was solved in the following manner:

(1). The initial orientation of the iceberg is assumed to be one of the stable positions.

(2). The surface of the iceberg below the waterline is given a melt displacement dz into the iceberg. In calculating dz, a melting time of 1800 seconds is assumed.

(3). Then the motion of the iceberg is modelled for 300 seconds to find the new orientation of the iceberg.

Except when roll-over occurs, the orientation of the iceberg with respect to the water surface changes only a little. The change in shape of the iceberg even for an hour of melting is extremely low. Therefore, this way of modelling the motion of the iceberg will not introduce any appreciable error.

This type of analysis is applied to each of the icebergs considered in this work, taking each stable position to be the initial orientation. The process of melting obviously changes the cross-sectional shape of the iceberg. Because of this change in shape, the orientation of the iceberg with respect to the waterline might also change. If the melting analysis is carried out for sufficiently longer time, then the shape of the iceberg might get changed to such an extent that it causes a roll-over.

In order to demonstrate this and thereby to establish that the algorithm developed in this work can model such a phenomenon, the cross-sectional shape of the iceberg as well as the orientation of the iceberg with respect to the waterline is plotted for different times. The melting analysis is carried out for sufficiently longer time so that the roll-over behaviour can be modelled.

An estimate of the time of roll-over is obtained, for the cases which exhibit the roll-over phenomenon. In other cases which do not exhibit the roll-over phenomenon, a possible reason for this behaviour is found out.

Figures 11a to 11d show the results of the melting analysis for the typical shape, when the initial orientation is taken to be the one shown in Figure 5b. The angular orientation corresponding to Figure 5b is 0.99233 rad. The angular orientations corresponding to Figures 11a to 11d are different

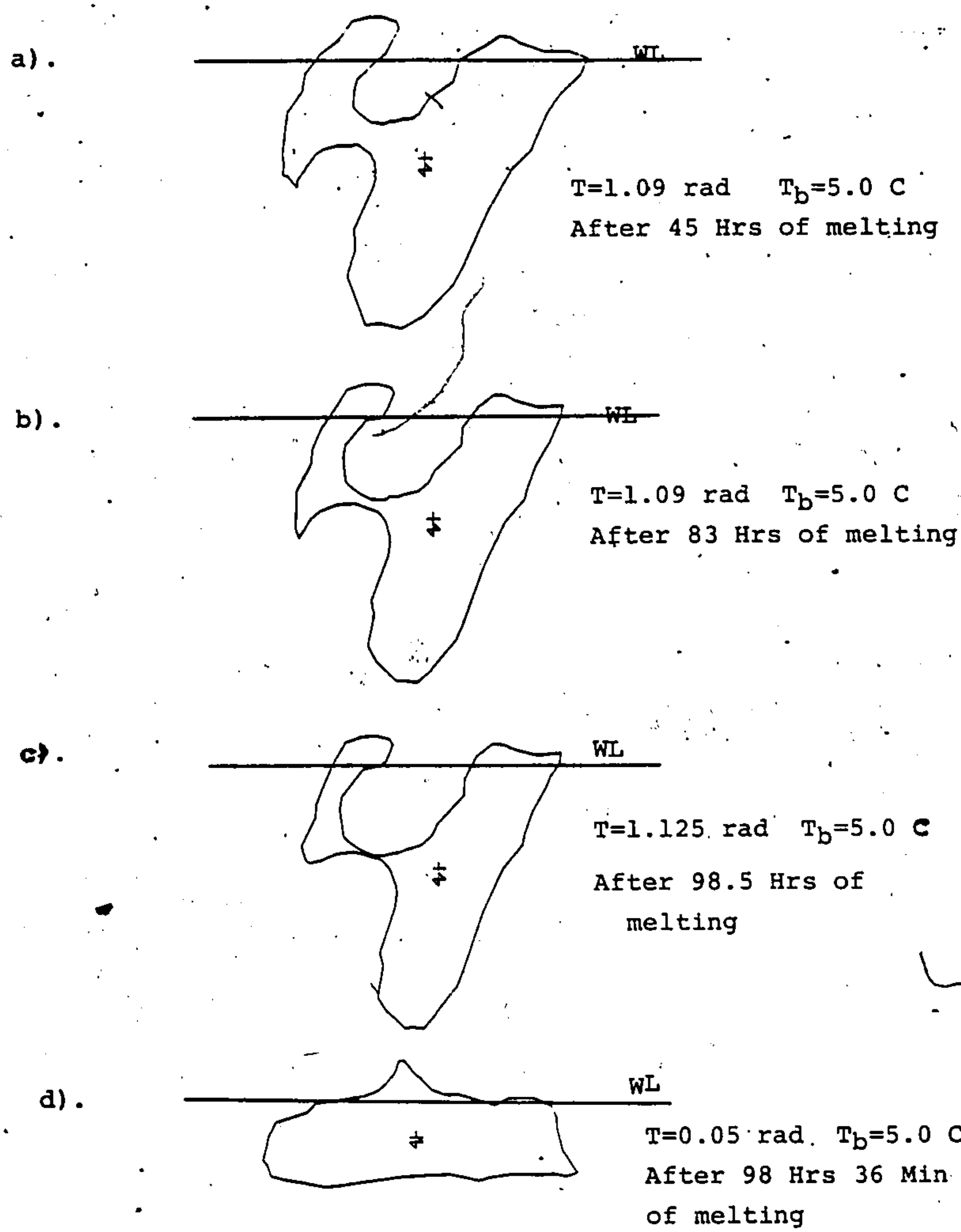


Figure 11. Illustration of Roll-over of the Typical iceberg (Initial Orientation Figure 5b)

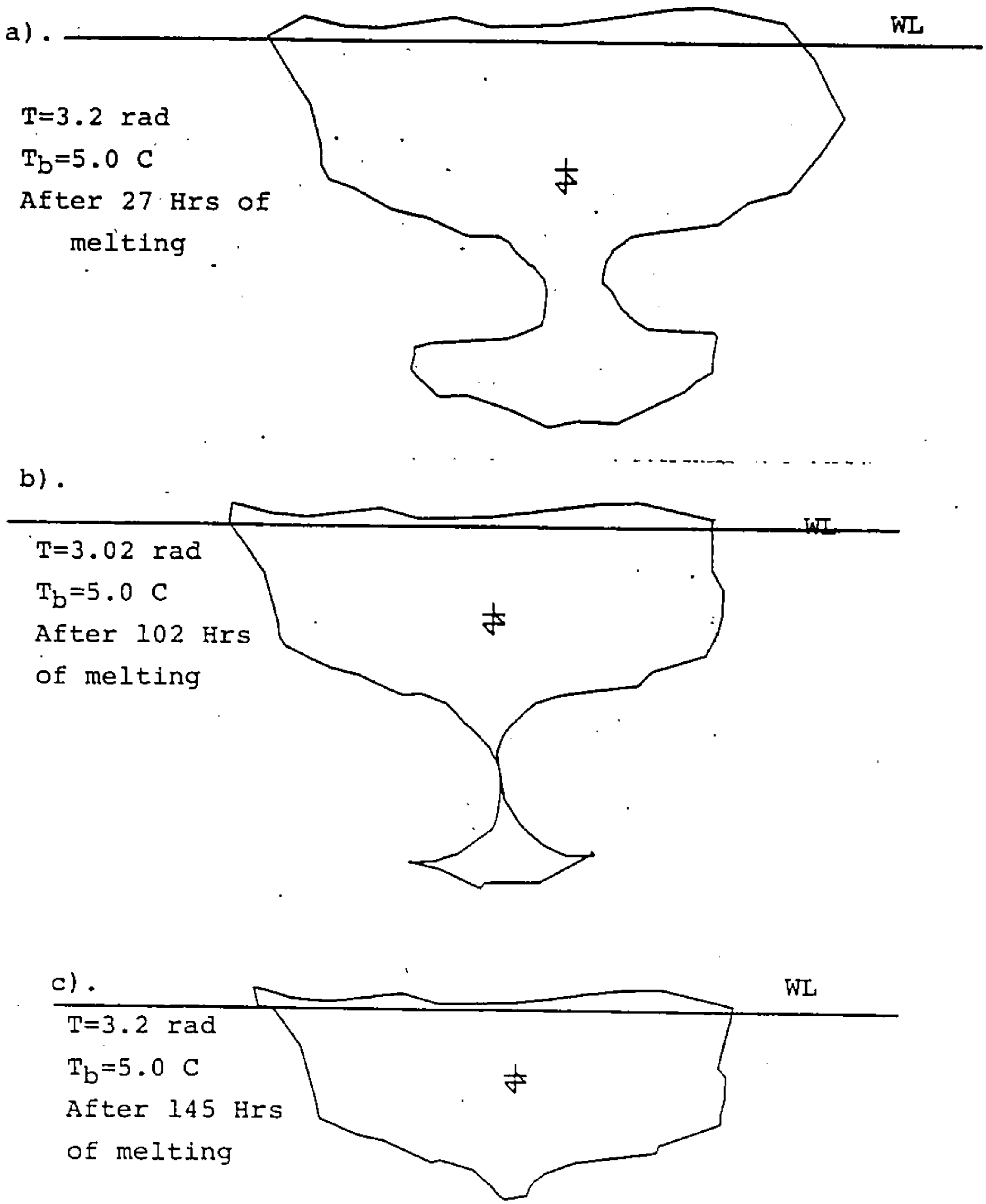


Figure 12. The Change in Cross-sectional shape of
the Typical Iceberg (Initial Orientation Figure5d)

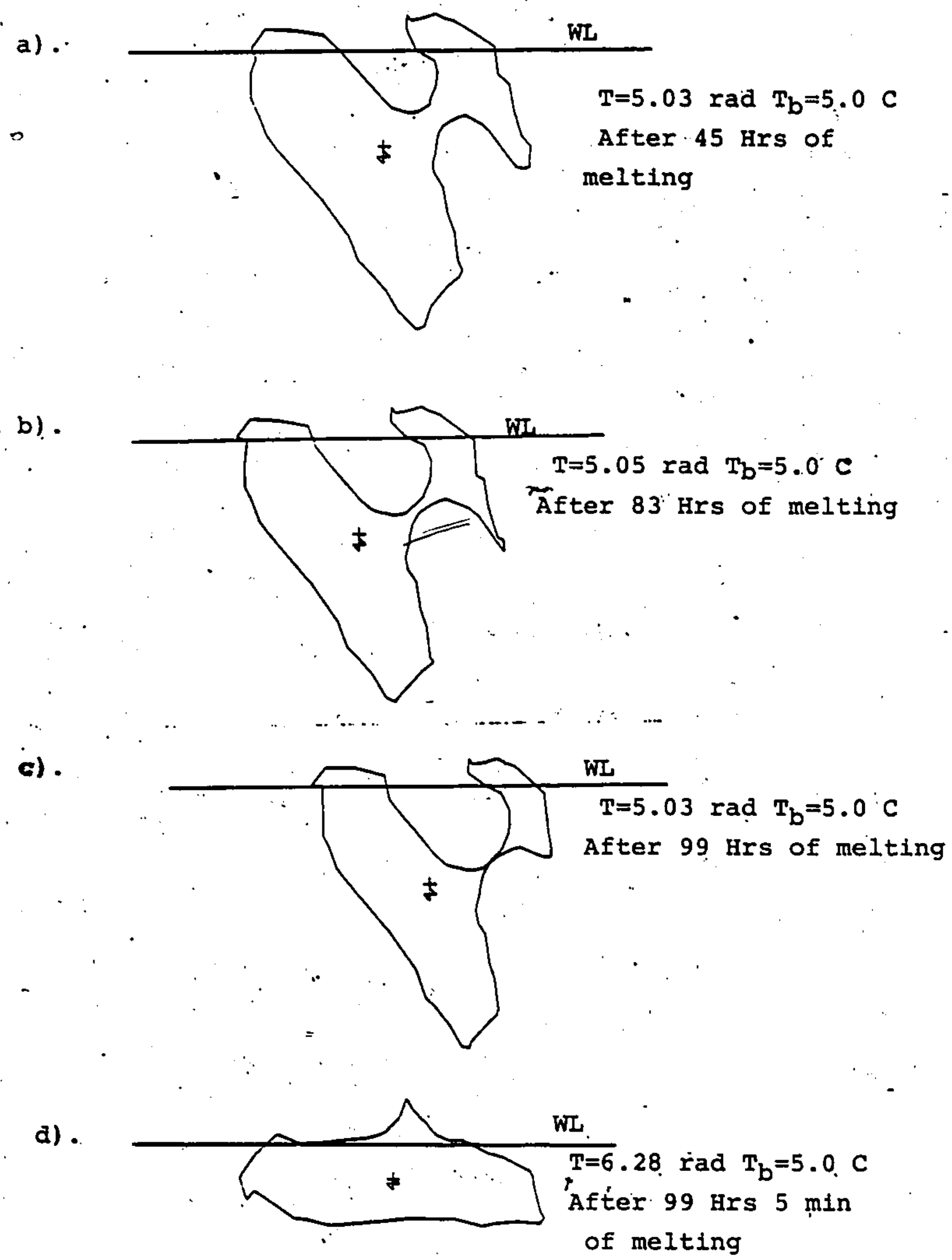


Figure 13. Illustration of Roll-over of the Typical Iceberg (Initial Orientation Figure 5f)

from 0.99233 rad. This change in angular orientation is caused because of melting. As shown in Figure 11c, (at time =98.5 hours) a small portion of the iceberg breaks away from the main iceberg. Due to this breaking away, the present floating position becomes unstable and hence the iceberg rolls over to a new stable equilibrium position, at 98 hours 36 minutes.

Figures 12a to 12c show the results of the melting analysis when the initial orientation is taken to be the one shown in Figure 5d. In this case also, a portion of the iceberg breaks away from the main block. However, in this case no roll over was observed. This is mainly due to the nature of the shape the iceberg attains after 145 hours of melting as shown in Figure 12c. This shape more closely resembles a rectangular shape, with thickness small compared to the length. Such rectangular shapes will not roll over, however long the melting time might be. This may be explained as follows. Since the thickness of the shape is quite small compared to its length, the effect of melting is essentially to reduce the thickness of the iceberg, with its shape remaining basically rectangular. It is also known that a rectangular iceberg with its thickness small compared to its length will float in stable position, when its length dimension is parallel to the waterline surface. In this case, the rectangular shape always floats with its length dimension parallel to the waterline surface. Melting does not change this

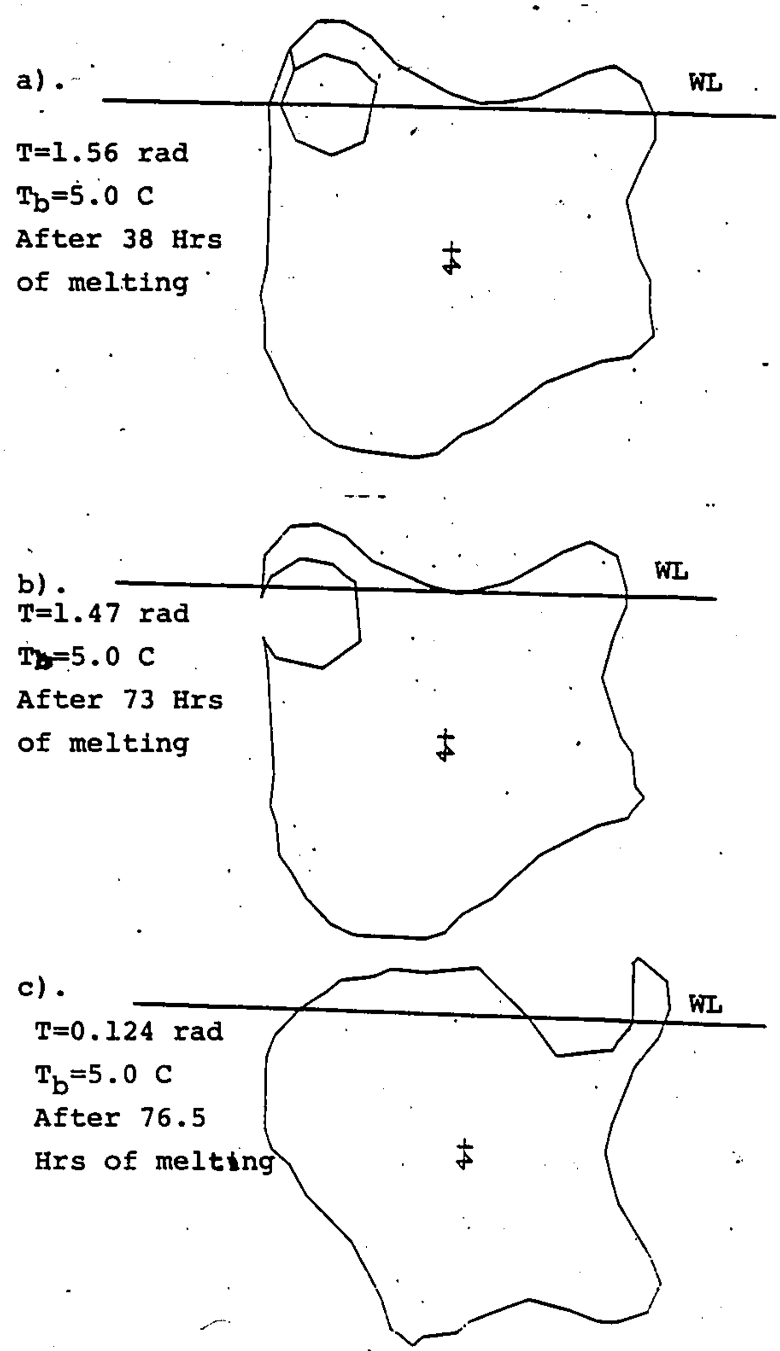
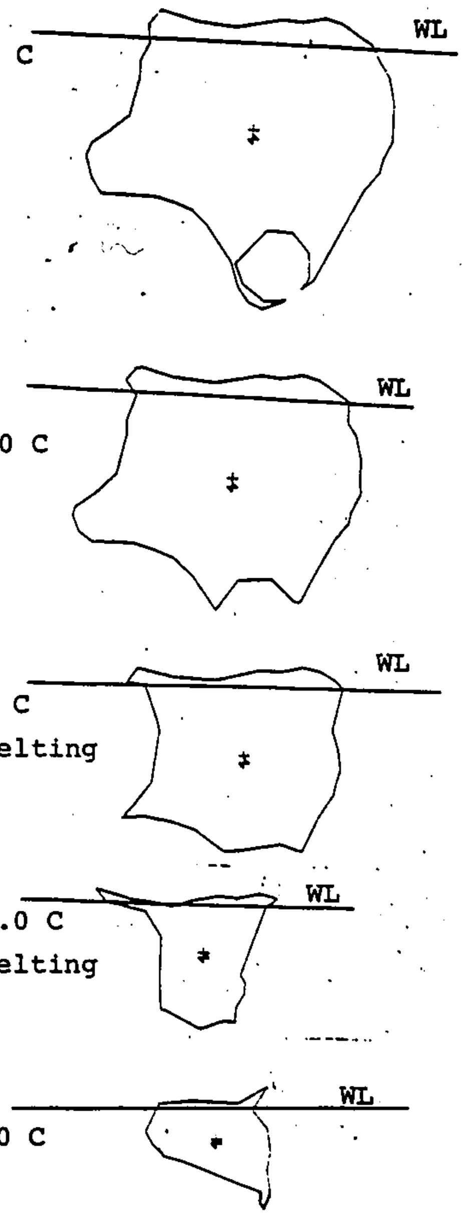


Figure 14. Illustration of Roll-over of the Irregular Shape with a Hole (Initial Orientation Figure6b)



a) T=4.25 rad T_b=5.0 C
After 38 Hrs of
melting

b) T=4.25 rad T_b=5.0 C
After 83 Hrs of
melting

d) T=4.25 rad T_b=5.0 C
After 218 Hrs of melting

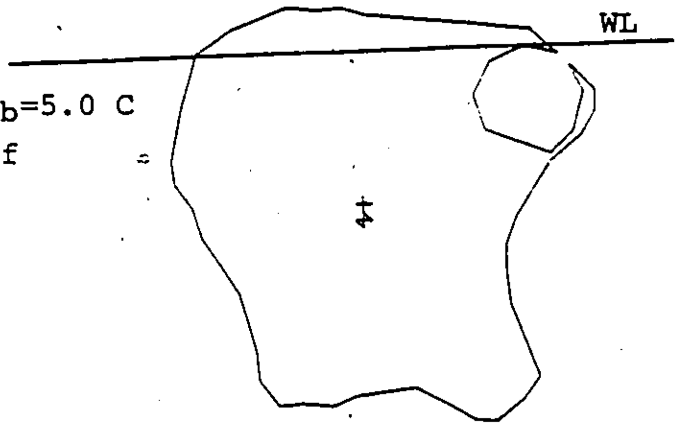
[illegible]) T=4.25 rad T_b=5.0 C
After 413 Hrs of melting

e) T=2.58 rad T_b=5.0 C
After 463 Hrs 6Min
of melting

Figure 15. Illustration of Roll-over of the Irregular Iceberg with a Hole (Initial Orientation Figure 6d)

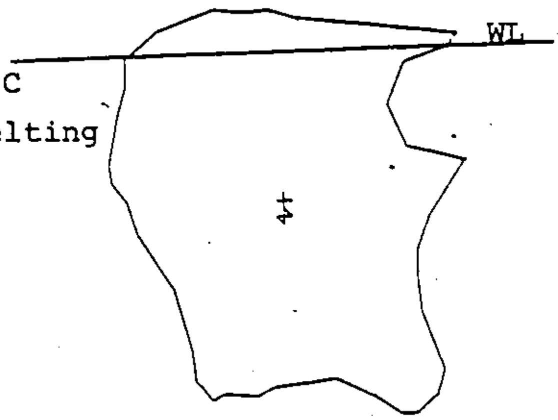
a).

T=6.17 rad T_b=5.0 C
After 38 Hrs of
melting



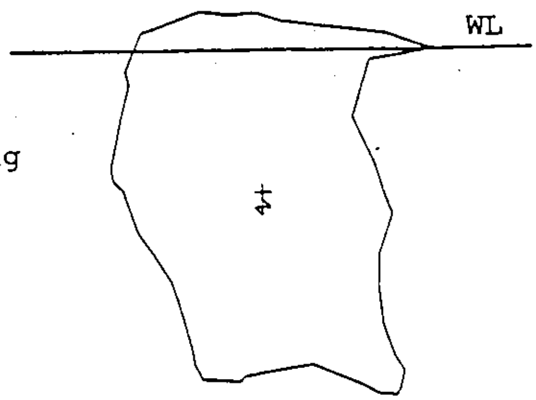
b).

T=6.2 rad T_b=5.0 C
After 76 Hrs of melting



C).

T=6.2 rad T_b=5.0 C
After 138 Hrs of melting



d).

T=7.7 rad T_b=5.0 C
After 139 Hrs of melting

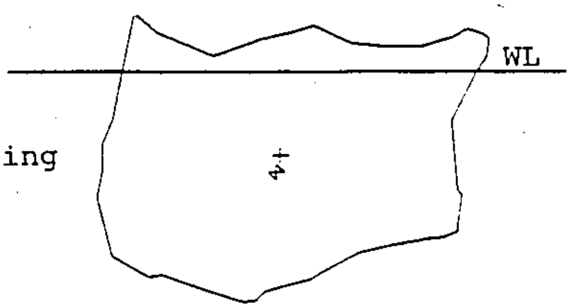


Figure 16. Illustration of Roll-over of the Irregular Iceberg with a Hole (Initial Orientation Figure6f)

condition. Therefore, no roll-over was observed.

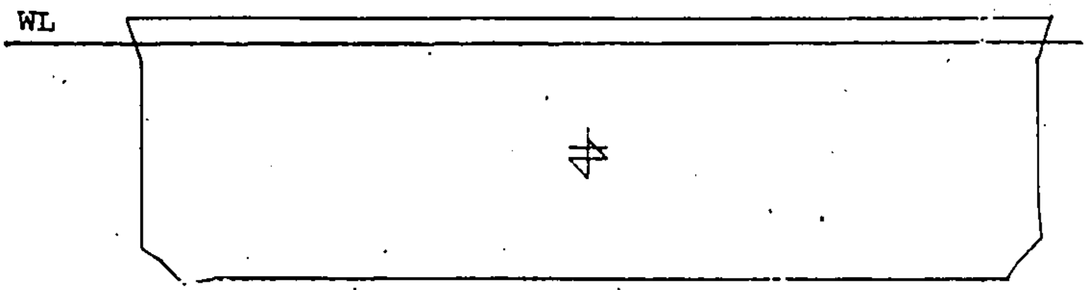
Figures 13a to 13d show the result of the melting analysis for the typical shape, when the initial orientation is taken to be the one shown in Figure 5f. In this case, after 99 hours of melting, a small portion of the iceberg breaks away from the main block. This breaking away renders the present floating position unstable and hence the iceberg rolls over to a new stable orientation.

This melting procedure was then applied to the irregular shape with a hole in it. Figures 14a to 14c show the change in shape of the iceberg along with its orientation with respect to the waterline, when the initial orientation is taken to be the one shown in Figure 6b.

Figures 15a to 15e show the results of the melting analysis, when the initial orientation is taken to be the one shown in Figure 6d. Even though the iceberg has a hole initially, the hole completely disappears after 83 hours of melting. The shape undergoes further changes and eventually rolls over at 463 hours and 6 minutes.

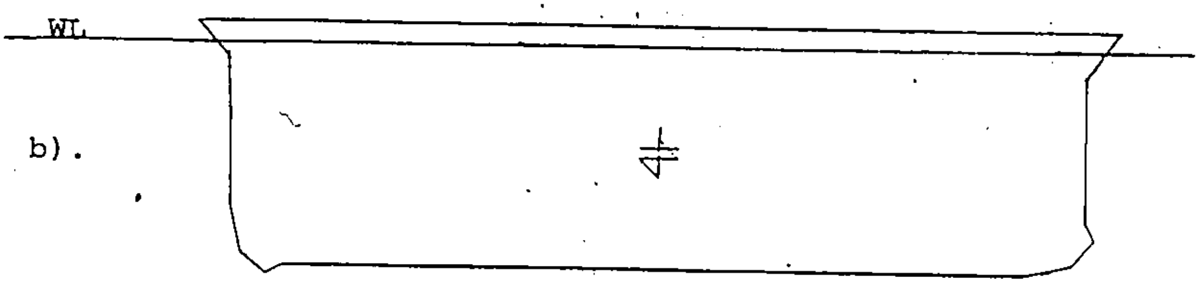
Figures 16a to 16d show the result of the melting analysis applied to the irregular shape with a hole, when the initial orientation is taken to be the one shown in Figure 6f.

a)



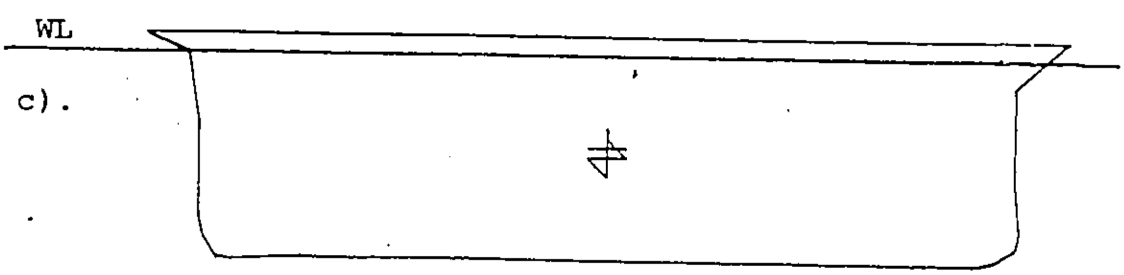
After 19 Hrs of melting $T=0.785$ rad $T_b=5.0$ C

b).



After 69 Hrs of melting $T=0.786$ rad $T_b=5.0$ C

c).



After 163 Hrs of melting $T=0.786$ rad $T_b=5.0$ C

Figure 17. The Change in Cross-sectional Shape of the Rectangular Iceberg (Initial Orientation Figure 7a)

The purpose of analysing this irregular shape with a hole is to show that the melting analysis developed in this work can be applied to an irregular shape with a hole in it.

Finally, Figures 17a to 17c show the result of the melting analysis applied to the rectangular shape. However, in this case, no roll-over was observed. This is because, the effect of melting is essentially to reduce the thickness of the iceberg, with its shape remaining basically rectangular. Since this shape floats with its length dimension parallel to the waterline surface, obviously no roll-over was observed.

Table 2 gives the roll-over time for different orientations along with how stable one position is compared to the other, for a particular shape. As can be noted from Table 2, the roll-over time does not depend on how stable one position is compared to the other for a particular shape.

From this analysis, the following conclusions are drawn.

(1). Roll over is dependent on the shape as well as on the initial orientation

(2). Roll over time is not dependent on how stable one position is compared to the other, for a particular shape.

TABLE 2

ROLL OVER TIME

INITIAL ORIENTATION	RELATIVE STABILITY	ROLL OVER TIME
Fig. 5b	II most stable	98 Hrs. 36 min.
Fig. 5d	most stable	-No Roll Over
Fig. 5f	III most stable	-99 Hrs. 6 min.
Fig. 6b	II most stable	76.5 Hrs.
Fig. 6d	III most stable	463 Hrs. 6 min.
Fig. 6f	most stable	139 Hrs.
Fig. 7b	-	No Roll Over

4. CONCLUSIONS AND RECOMMENDATIONS

1. A general algorithm in the form of a computer program has been developed to model the floating characteristics of a two dimensional irregularly shaped iceberg floating on water.

2. The algorithm has three important parts. The first part of the algorithm helps find out the various stable orientations of a two dimensional irregular iceberg. Results show that a two dimensional irregular iceberg has several stable orientations and several unstable ones.

3. From a particular orientation, as the iceberg is rotated through 2π radians, stable and unstable positions occur one after the other. The number of stable positions must be equal to the number of unstable ones. However, these two conclusions are not true, if an intervening neutral equilibrium position occurs.

4. Certain stable positions are more stable than other stable ones, for a particular shape.

5. The second part of the algorithm helps model the motions of a two dimensional irregularly shaped iceberg.

6. The validity of the dynamic analysis is established by using it to predict the motions of the iceberg from a

particular orientation for the case of no melting. It is found that this brings the iceberg to one of the stable positions found from the stability analysis. It is also found that the stable position to which the iceberg is brought is dependent on the initial orientation.

7. The third part of the algorithm models the motions of the iceberg arising due to melting. Using this, it is established that the roll over time is not dependent on how stable one position is compared to the other, for a particular shape.

8. Roll over is dependent on the shape as well as on the initial orientation.

4.1 AREAS OF FUTURE RESEARCH:

1. As mentioned in the introduction, icebergs are in fact three dimensional irregularly shaped masses. Therefore, it is recommended that the present analysis be extended to the three dimensional case.

2. It may be advantageous if an unconditionally stable numerical scheme can be designed for solving the equations of motion. The reason for this is that such a scheme might reduce the amount of computer time.

3. Since icebergs are quite irregular, the natural

convection taking place around them is much more complicated in nature. Because of this, the heat transfer coefficient might be different along different sections of the iceberg. The present melting model must be improved to incorporate this phenomenon.

4. The theoretical conclusions drawn in this work should be verified by means of an experiment. The experiment can be carried out with a small ice block and the results predicted by this analysis can be verified, with appropriate modifications being made to the melting model.

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TABLE 3

Co-ordinate Points Describing the Typical Shape

INDEX NO	X-CO-ORD	Y-CO-ORD
1	0.0	40.0
2	10.0	39.0
3	20.0	40.0
4	30.0	35.0
5	40.0	31.0
6	50.0	30.0
7	55.0	25.0
8	58.0	20.0
9	59.5	13.0
10	58.0	10.5
11	50.0	10.2
12	45.0	10.0
13	40.0	10.0
14	30.0	10.0
15	26.0	9.0
16	22.0	8.5
17	21.8	8.0
18	21.6	7.5
19	21.4	7.0
20	21.3	6.5
21	20.9	4.0
22	20.6	2.0
23	20.3	1.0
24	20.0	0.0
25	21.0	-1.0
26	22.0	-3.0
27	24.0	-4.0
28	25.0	-5.0
29	26.0	-6.0
30	28.0	-8.0
31	29.0	-10.0
32	30.0	-11.0
33	40.0	-12.0
34	50.0	-17.0
35	60.0	-20.0
36	70.0	-26.0
37	78.0	-30.0
38	80.0	-35.0
39	80.0	-40.0
40	82.0	-50.0
41	88.0	-60.0
42	90.0	-70.0
43	80.0	-75.0
44	70.0	-72.0
45	60.0	-71.0
46	50.0	-72.0
47	40.0	-73.0
48	30.0	-70.0
49	20.0	-70.0
50	10.0	-70.0
51	0.0	-71.0
52	-10.0	-72.0
53	-20.0	-73.0
54	-30.0	-73.0
55	-40.0	-70.5
56	-50.0	-68.0
57	-60.0	-60.0
58	-62.0	-60.0
59	-65.0	-50.0
60	-68.0	-40.0
61	-60.0	-30.0
62	-50.0	-20.0
63	-40.0	-18.0
64	-30.0	-12.0
65	-20.0	-11.0
66	-10.0	-10.0
67	- 8.0	-10.0
68	- 7.0	- 9.5
69	- 6.0	- 8.5
70	- 5.0	- 8.0
71	- 4.0	- 8.0
72	- 3.0	- 6.0
73	- 2.0	- 5.0
74	0.0	- 3.0
75	- 3.0	0.0
76	- 5.0	4.0
77	- 7.0	6.0
78	- 9.0	8.0
79	-10.0	9.0
80	-20.0	10.0
81	-30.0	12.0
82	-35.0	16.0
83	-32.0	16.0
84	-30.0	21.0
85	-28.0	27.0
86	-24.0	29.0
87	-20.0	32.0
88	0.0	40.0

TABLE 4

Co-ordinate Points Describing the Irregular Shape with a Hole

INDEX NO	X-CO-ORD	Y-CO-ORD
1	0.0	60.0
2	20.0	60.0
3	40.0	60.0
4	60.0	60.0
5	80.0	50.0
6	72.0	48.0
7	60.0	50.0
8	50.0	45.0
9	45.0	35.0
10	48.0	25.0
11	68.0	17.0
12	70.0	20.0
13	75.0	25.0
14	78.0	37.0
15	72.0	48.0
16	80.0	50.0
17	90.0	40.0
18	90.0	30.0
19	85.0	20.0
20	75.0	10.0
21	70.0	0.0
22	65.0	-10.0
23	62.0	-20.0
24	63.0	-30.0
25	65.0	-40.0
26	70.0	-50.0
27	75.0	-60.0
28	78.0	-70.0
29	72.0	-80.0
30	60.0	-90.0
31	50.0	-90.0
32	40.0	-85.0
33	30.0	-80.0
34	10.0	-85.0
35	0.0	-90.0
36	-10.0	-90.0
37	-20.0	-92.0
38	-30.0	-90.0
39	-32.0	-80.0
40	-34.0	-70.0
41	-38.0	-60.0
42	-42.0	-50.0
43	-50.0	-40.0
44	-58.0	-30.0
45	-62.0	-20.0
46	-70.0	-10.0
47	-72.0	0.0
48	-71.0	10.0
49	-70.0	20.0
50	-68.0	30.0
51	-62.0	40.0
52	-50.0	50.0
53	-40.0	55.0
54	-30.0	60.0
55	-20.0	60.0
56	-10.0	62.0
57	0.0	60.0

TABLE 5

Co-ordinate Points Describing the Rectangular Shape

INDEX NO	X-CO-ORD	Y-CO-ORD
1.	80.0	-20.0
2.	75.0	-25.0
3.	70.0	-30.0
4.	65.0	-35.0
5.	60.0	-40.0
6.	55.0	-45.0
7.	50.0	-50.0
8.	45.0	-45.0
9.	40.0	-40.0
10.	35.0	-35.0
11.	30.0	-30.0
12.	25.0	-25.0
13.	20.0	-20.0
14.	15.0	-15.0
15.	10.0	-10.0
16.	5.0	- 5.0
17.	0.0	0.0
18.	- 5.0	5.0
19.	-10.0	10.0
20.	-15.0	15.0
21.	-20.0	20.0
22.	-25.0	25.0
23.	-30.0	30.0
24.	-35.0	35.0
25.	-40.0	40.0
26.	-45.0	45.0
27.	-50.0	50.0
28.	-45.0	55.0
29.	-40.0	60.0
30.	-35.0	65.0
31.	-30.0	70.0
32.	-25.0	75.0
33.	-20.0	80.0
34.	-15.0	75.0
35.	-10.0	70.0
36.	- 5.0	65.0
37.	0.0	60.0
38.	5.0	55.0
39.	10.0	50.0
40.	15.0	45.0
41.	20.0	40.0
42.	25.0	35.0
43.	30.0	30.0
44.	35.0	25.0
45.	40.0	20.0
46.	45.0	15.0
47.	50.0	10.0
48.	55.0	5.0
49.	60.0	0.0
50.	65.0	- 5.0
51.	70.0	-10.0
52.	75.0	-15.0
53.	80.0	-20.0

TABLE 6

Moment Arm and Angular Orientation for the Typical Shape

SERIAL NO	T	A
1	0.1000	-0.0507
2	0.3000	0.1319
3	0.1555	-0.0214
4	0.1757	-0.0063
5	0.1840	0.0005
6	0.1834	0.0000
7	0.3834	0.3207
8	0.5834	1.0655
9	0.7834	1.6501
10	0.9834	0.1140
11	1.1835	-1.8082
12	0.9953	-0.0388
13	0.9912	0.0149
14	0.9923	0.0000
15	1.1923	-1.8631
16	1.3923	-2.6412
17	1.5923	-2.1882
18	1.7923	-1.1256
19	1.9923	-0.0171
20	2.1923	1.0606
21	1.9955	0.0004
22	1.9954	0.0000
23	2.1954	1.0765
24	2.3954	2.0540
25	2.5954	2.9322
26	2.7954	3.6197
27	2.9954	3.3505
28	3.1954	-0.1337
29	3.1877	0.0494
30	3.1898	0.0008
31	3.1899	0.0000
32	3.3899	-3.0179
33	3.5899	-3.0386
34	3.7899	-2.5736
35	3.9899	-1.8464
36	4.1899	-0.9686
37	4.3899	-0.0212
38	4.5899	0.9577

SERIAL NO	T	A
39	4.3942	-0.0003
40	4.5943	0.9794
41	4.7943	1.9414
42	4.9943	2.2508
43	5.1943	1.4370
44	5.3943	-0.1028
45	5.3809	0.0050
46	5.3815	0.0000
47	5.5815	-1.6754
48	5.7815	-1.0996
49	5.9815	-0.3978
50	6.1815	-0.1177
51	6.3815	-0.0512

TABLE 7

Moment Arm and Angular Orientation for the Irregular Shape with a hole

SERIAL NO	T	A
1	0.1000	-3.1784
2	0.3000	-2.7455
3	0.5000	-2.1871
4	0.7000	-1.6108
5	0.9000	-0.9683
6	1.1000	-0.2352
7	1.3000	0.5163
8	1.1626	0.0012
9	1.1623	0.0000
10	1.3623	0.7433
11	1.5623	1.4133
12	1.7623	2.0627
13	1.9623	2.6835
14	2.1623	3.1820
15	2.3623	3.4462
16	2.5623	3.1106
17	2.7623	-1.1598
18	2.7080	0.4290
19	2.7226	0.0008
20	2.7227	0.0000
21	2.9227	-3.2550
22	3.1227	-3.4055
23	3.3227	-3.0974
24	3.5227	-2.5821
25	3.7227	-1.9726
26	3.9227	-1.3544
27	4.1227	-0.6432
28	4.3227	0.1250
29	4.2901	-0.0010
30	4.2904	0.0000
31	4.4904	0.7635
32	4.6904	1.4564
33	4.8904	2.0729
34	5.0904	2.6727
35	5.2904	3.1570
36	5.4904	3.4204
37	5.6904	3.1063

SERIAL NO	T	A
38	5.8904	-0.8997
39	5.8555	0.3619
40	5.8583	0.0004
41	5.8584	0.0000
42	6.0584	-3.1715
43	6.2584	-3.3323
44	6.4584	-3.0369

TABLE 8

Moment Arm and the Angular Orientation for the Rectangular Shape

SERIAL NO	T	A
1	0.1000	-0.9872
2	0.3000	-1.2554
3	0.5000	-0.8917
4	0.7000	-0.2683
5	0.9000	0.5094
6	0.7690	-0.0126
7	0.7722	-0.0005
8	0.7723	0.0000
9	0.9723	0.8060
10	1.1723	1.5939
11	1.3723	1.0825
12	1.5723	-0.0589
13	1.5620	-0.0040
14	1.5612	0.0000
15	1.7612	-0.7910
16	1.9612	-1.9908
17	2.1612	-1.7335
18	2.3612	-0.6068
19	2.5612	0.3966
20	2.4822	0.0459
21	2.4718	-0.0066
22	2.4731	0.0000
23	2.6731	0.7965
24	2.8731	1.0311
25	3.0731	1.0547
26	3.2731	1.0057
27	3.4731	0.9417
28	3.6731	1.0319
29	3.8731	1.2458
30	4.0731	0.7874
31	4.2731	-0.1289
32	4.2450	0.0293
33	4.2502	0.0010
34	4.2504	0.0000
35	4.4504	-1.3403
36	4.6504	-1.8137
37	4.8504	-1.4140
38	5.0504	-0.8872
39	5.2504	-0.2616
40	5.4504	0.3039
41	5.5357	0.5216
42	5.7357	0.9763
43	5.9357	1.1122
44	6.1357	0.2658
45	6.3357	-0.8159
46	6.1849	-0.0583
47	6.1733	0.0210
48	6.1763	0.0000
49	6.3763	-0.9654

TABLE 9

Consecutive Amplitudes of the Test Piece

SERIAL NO	X1	X2	C1
1.	6	4	0.06432
2.	9	5	0.09310
3.	12	7	0.08550
4.	15	8	0.09950
5.	17	9	0.10060
6.	19	10	0.10160

APPENDIX A

AREA CALCULATION:

In order that one can deal extensively with the floating characteristics of a two dimensional object having irregular cross-section, it becomes necessary to develop a general algorithm to determine the values of buoyancy and weight forces as well as their points of action. The weight of the iceberg per unit length is the product of the cross-sectional area and the specific weight of the iceberg. The force of buoyancy per unit length is the product of the cross-sectional area below the waterline and the specific weight of water. This in turn necessitates the development of a general and easy algorithm to determine the area and the centroid of an irregular cross-section. Two points need an emphasis here. Firstly, the algorithm must be quite general so that it can be adapted to any irregular cross-section and secondly it must be adaptable to computers.

If the algorithm becomes adaptable to computers, then it can be developed in the form of a computer program which will calculate the necessary quantities with the shape of the irregular cross-section having been supplied to the program.

With such an idea in mind, the following algorithm has been developed. It is assumed here that the irregular shape has been defined as co-ordinate points with respect to an arbitrarily chosen Cartesian co-ordinate system as shown in Figure 18.

A careful look at Figure 18 will reveal that the entire irregular area can be approximated by many triangular areas. Naturally the sum of these triangular areas will constitute a good approximation to the irregular area only if the points describing the irregular shape are close to one another so that the curve joining the two adjacent points can be approximated by a straight line. Thus closely spaced points need be specified in regions where the shape has a strong curvature. Then the area of the irregular shape is equal to the sum of the areas of the triangles. The problem now boils down to finding these triangular areas with the help of a computer.

The input to the computer program is the co-ordinate points defining the irregular shape with respect to an arbitrarily chosen Cartesian co-ordinate system. These points are input to the program in a particular order, (i.e.) starting from a particular point, the irregular shape is traced clockwise till the starting point is once again reached. During this tracing the co-ordinate points are input as and when they are traced and in that order, including the starting point once again.

Further development of the algorithm needs an extensive reference to the Figure 18. (A separate nomenclature is maintained for this section of the report). The values of a and b can be computed as follows:

$$a = \sqrt{x_{i-1}^2 + y_{i-1}^2}$$

$$b = \sqrt{x_i^2 + y_i^2}$$

It is clear from this that a and b represent the distances of points i-1 and i from the origin. The value of $d\theta_i$ is computed from the following expression.

$$d\theta_i = \mathrm{ATAN2}\left(\frac{y_{i-1}}{x_{i-1}}\right) - \mathrm{ATAN2}\left(\frac{y_i}{x_i}\right) \qquad \ldots\mathrm{A1}$$

Hence the area of the triangle can be computed by

$$dA_i = \tfrac{1}{2}\, ab \sin d\theta_i \qquad \ldots\mathrm{A2}$$

Then the area of the irregular shape is given by

$$A_R = \sum_{i=2}^{N} dA_i$$

where N is the total number of points.

The centroid of this area can be found by the following procedure. The moments of the entire area about the X and Y axes are given by

$$M_x = \sum_{i=2}^{N} dA_i\, y_{ci}$$

$$M_y = \sum_{i=2}^{N} dA_i\, x_{ci}$$

where x_{ci} and y_{ci} are the co-ordinates of the centroid of the triangle.

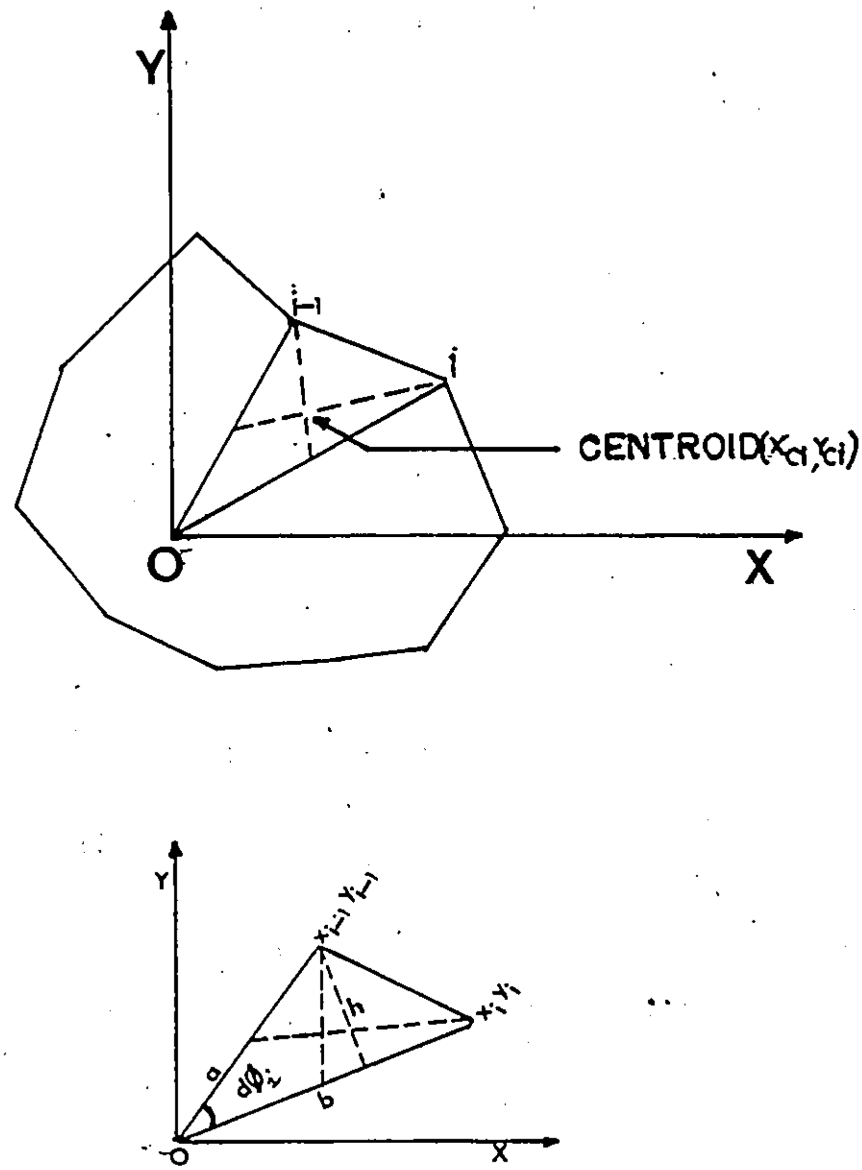

Figure 18. A Two Dimensional Irregular Cross-section (Case 1)

From this the co-ordinate points of the centroid of the entire area can be determined as follows:

$$x_a = M_y/A_R$$

$$y_a = M_x/A_R$$

The values of M_x and M_y necessitates determination of the values of (x_{ci}, y_{ci}) the co-ordinate points of the centroid of each triangle. The procedure used to determine the co-ordinate points of the centroid of each triangle is as follows.

The centroid of a triangle is the point of intersection of its medians. Therefore, if the equations for two of its medians are known, then its centroid is the point of intersection of the two straight lines. Without loss of generality, the two medians of the triangle can be represented by straight lines $y=m_{i-1}x-d_{i-1}$ and $y=m_i x+d_i$. The values of m_{i-1} and m_i can be found from

$$m_{i=1} = \frac{y_{i-1}-y_i/2}{x_{i-1}-x_i/2} \quad ; \quad m_i = \frac{y_i-y_{i-1}/2}{x_i-x_{i-1}/2}$$

Since the above straight lines pass through the points (x_{i-1}, y_{i-1}) and (x_i, y_i) the values of d_{i-1} and d_i can be evaluated by

$$d_{i-1} = y_{i-1} - m_{i-1}\, x_{i-1}; \quad d_i = y_i - m_i x_i$$

Both the straight lines pass through the centroid $(x_{ci} y_{ci})$. The values of x_{ci} and y_{ci} are determined by

$$x_{ci} = \frac{d_i - d_{i-1}}{m_{i-1} - m_i} \quad ; \quad y_{ci} = m_{i-1}\, x_{ci} + d_{i-1}$$

The above algorithm can be conveniently cast in the form of a computer program, which taking the co-ordinate points as input, will calculate the area and the co-ordinate points of the centroid of the area.

At first sight, this algorithm may appear to have some serious drawbacks especially for shapes such as the one shown in Figure 19. For the shape shown in Figure 19, it may appear that the area calculated by this procedure will have some unwanted area (the shaded portion a) also. In Figure 19, .the points 1,2,3,4,5,6,7 describe the irregular shape.

The following will reveal that the unwanted area will not be encountered in the course of the calculation. As described in this section of the report, the total area of the irregular shape is the algebraic addition of the triangular areas as calculated by equation A2. If some triangular areas become negative then those areas will be subtracted from the total area. The criteria which decides whether a triangular area is positive

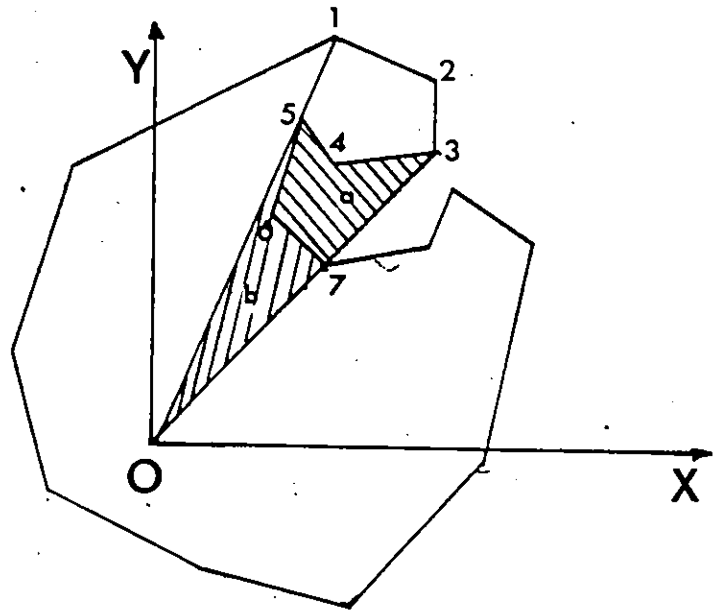

Figure 19. A Two Dimensional Irregular Cross-section (Case 2)

or negative is the angle $d\theta_i$ which is calculated from equation A1. If $d\theta_i$ is positive then $\sin d\theta_i$ will be positive. If $d\theta_i$ is negative, then $\sin d\theta_i$ will be negative and hence the area will become negative.

This way of calculating the value of $d\theta_i$ in a computer allows the value of $d\theta_i$ to become positive or negative depending upon whether the angle subtended between the line joining the point (x_{i-1}, y_{i-1}) and the origin and the X-axis is greater than or less than the angle subtended between the line joining the point (x_i, y_i) and the origin and the X-axis.

Further analysis needs an extensive reference to Figure 19. As the triangular areas are calculated from point 1 to point 3, the value of $d\theta_i$ is greater than zero and hence dA_i is also positive. From the Figure 19, it is quite clear that these triangular areas include the undesired portion also.(the shaded portion a). Moving from point 3 to point 5, the value of $d\theta_i$ will become negative and hence dA_i is negative. These triangular areas calculated from 3 to 5 subtract the undesired ones from the total area. However, they not only subtract the undesired ones but some desired portion of the area also (shaded portion b). In moving from 5 to 7, the value of $d\theta_i$ will become positive and hence dA_i is also positive. These triangular areas calculated from point 5 to 7 will add the area of the shaded portion b to the total area. Therefore, the algorithm developed does not have any drawbacks, as long as the points are sufficiently close to one another in regions where the shape has a strong curvature.

The following demonstrates that the above procedure could be used to find the area and centroid of an irregular cross-section having a hole in it. Consider Figure 20. This shows that both the irregular cross-section as well as the location and the profile of the hole are defined as co-ordinate points with respect to a particular co-ordinate system. It should be kept in mind that the hole can be irregular in shape. The area and hence the centroid of this cross-section can be calculated as follows. If one imagines that the area be cut with the aid of a pair of scissors (this to produce a straight line cut) from point 1 to point 2, then the Figure 20 will look as shown in Figure 20a.

Now if one orders the points as shown in the Figure 20a, then no problem will be encountered to calculate the area and the centroid. The same procedure could be used, with no modifications.

The subroutine CENTRO in Appendix C calculates the area and the co-ordinate points of the centroid of any two

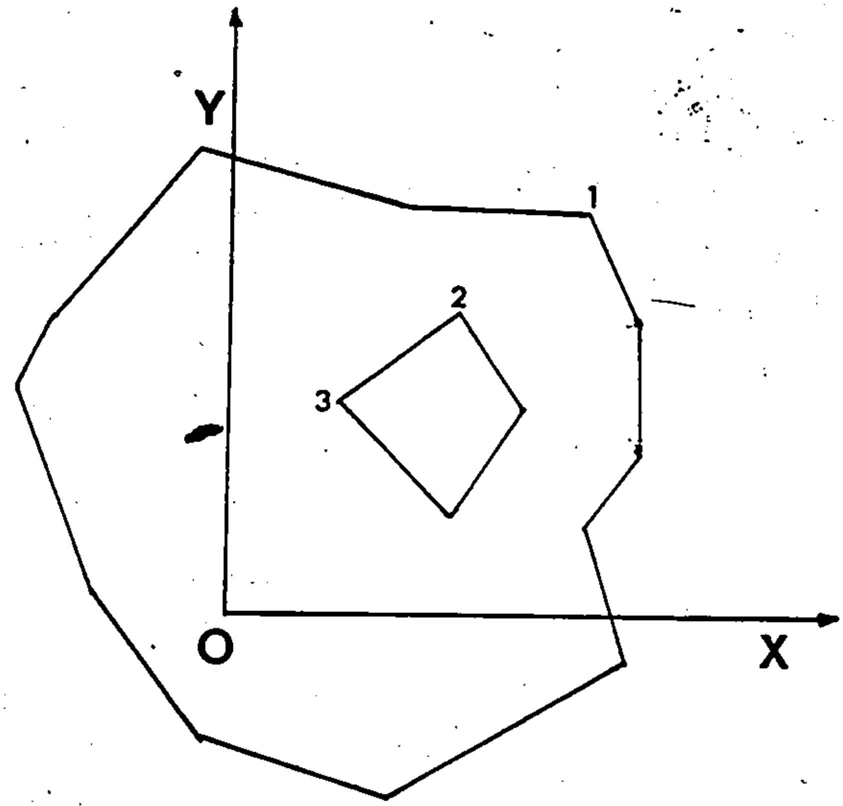

Figure 20.

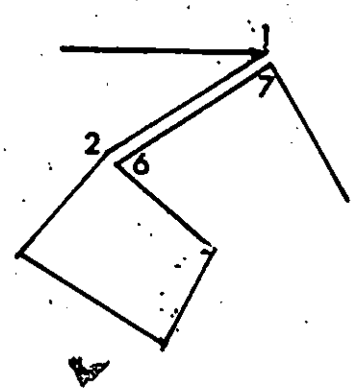

Figure 20a.

Figure 20. A Two Dimensional Irregular Cross-section with a Hole

dimensional irregular shape. As can be seen from the parameter list, the parameters passed are x,y, the co-ordinate of each point and N, the number of points. The subroutine CENTRO calculates the area, ACG and the co-ordinate points of the centroid of the irregular cross-section (XCG,YCG). It also gives each triangular area DAR and the co-ordinate points of the centroid of each triangle (XB,YB). In equation 2, the value of I, the mass moment of inertia about its centre of gravity is needed. For this purpose, a subroutine NEWICE is written. This subroutine NEWICE changes the origin of the co-ordinate system to the centre of gravity of the irregular area. Once this is done, the subroutine CENTRO is once again used to calculate each triangular area DAR and the co-ordinate points of the centroid of each triangular area (XB,YB) with respect to the new origin. Then the value of I_1 is computed as follows:

$$I_1 = \sum_{i=2}^{N} dA_i (x_{ci}^2 + Y_{ci}^2)$$

If I_1 is multiplied by the density of ice, then $I = I_1 * \rho$ gives the mass moment of inertia of the iceberg about its centre of gravity. (ρ is the density of ice)

BUOYANCY CALCULATION:

In order to calculate the force of buoyancy, the area of the iceberg below the water line should be computed. This is done as follows: A right-handed rectangular Cartesian co-ordinate system is attached to the water surface in such a way that the abscissa of that co-ordinate system is along the water surface. The calculation of the area below the waterline needs the orientation of the iceberg with respect to the waterline to be specified. There are two variables that determine the orientation of the iceberg with respect to the waterline. They are

(1). the angular orientation of the iceberg with respect to the waterline. This is represented by a quantity T

(2). how much of the portion of the iceberg is below the waterline. This is determined by the quantity Y ,, the distance of the centre of gravity of the iceberg from the waterline. Consider Figure 21. In this Figure, the x'-y' co-ordinate system represents the waterline co-ordinate system, with x'-axis running along the water surface.

If it is assumed that the shape of the iceberg is defined with respect to the x-y co-ordinate system, with the origin of this co-ordinate system lying on the centre of gravity of the iceberg, then the x-y co-ordinate system will look as shown in Figure 21, with respect to the waterline co-ordinates. The value of Y is referred to from the x' line. A positive Y means that the centre of gravity of the iceberg is above the

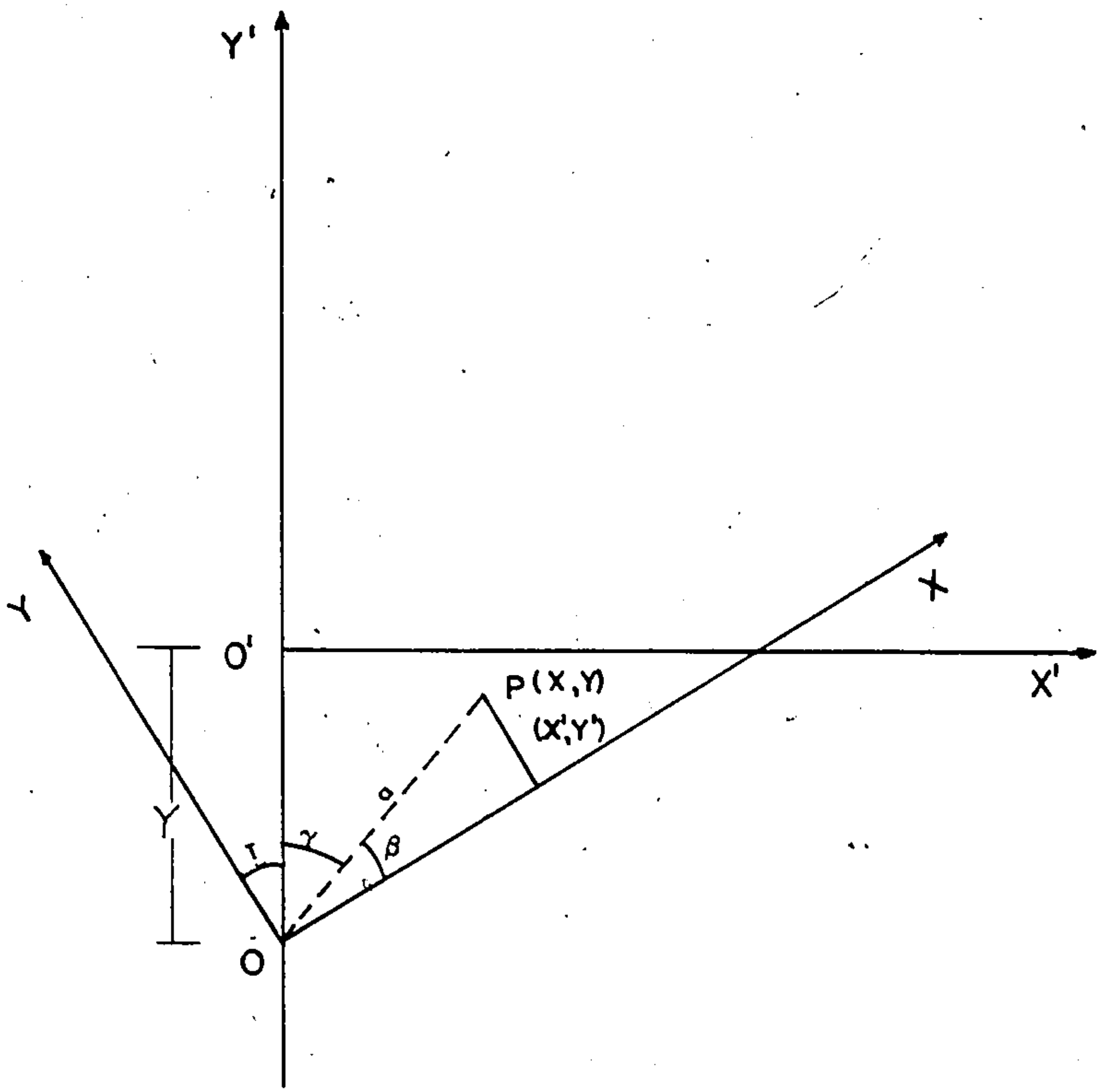

Figure 21. Illustration of the Buoyancy Calculation

waterline and a negative Y means that the centre of gravity of the iceberg is below the waterline. A positive T means that the iceberg is rotated in an anticlockwise direction and a negative T means that the iceberg is rotated in a clockwise direction. In order to calculate the cross-sectional area of the iceberg below the waterline, the co-ordinate points describing the iceberg are temporarily transferred from the x-y co-ordinate system to the x'-y' co-ordinate system. The points describing the iceberg that lies below the waterline will have a negative ordinate value with respect to the x'-y' co-ordinate system. These points (points that lie below the waterline) along with the points representing the intersection of the waterline with the iceberg are fed to the CENTRO routine in a clockwise ordering. The CENTRO routine then calculates the area and the centroid of the area below the waterline. In this work, this area and the centroid of this area are calculated with respect to the x-y co-ordinate system. To determine the value of the moment arm A, the following method is used: The moment arm A represents the perpendicular distance between the vertical line passing through the centroid of the whole area and the vertical line passing through the centroid of the area below the waterline. The co-ordinate points of the centroid of the whole area and the centroid of the area below the waterline are transformed temporarily to the x'-y' co-ordinate system. Then the value of A is obtained by subtracting the abscissa of the centroid of the whole area from the abscissa of the centroid of the area below the waterline. This means that, if the centre of buoyancy lies to the right of centre of gravity, then A will be positive and if it is to the left, it will be negative.

APPENDIX B

DAMPING COEFFICIENT CALCULATION:

The consecutive peaks shown in Table 9 are used to calculate the damping ratio C_d. The following shows the calculation procedure.

From the ratio of X1/X2, the logarithmic decrement δ can be calculated. Using the relationship (12) between δ and C_1, the value of C_1 is calculated.

MODEL CALCULATION:-

Observation No.2 is taken for example.

$$\delta = \ln(X1/X2) = \frac{2\pi\ C_1}{\sqrt{1-C_1^{\ 2}}}$$

$$\therefore \quad \ln(9/5) = \frac{2\pi\ C_1}{\sqrt{1-C_1^{\ 2}}}$$

$$0.345 - 0.345\ C_1^{\ 2} = 4\pi^2\ C_1^{\ 2}$$

$$\rightarrow C_1 = 0.0931$$

APPENDIX C

```
C
C
CCNFUTER FRCGRAMS
C
C
      SLBFCLTINE CENTRO(X,Y,XB,YE,DAR,N,AREA,XBAR,YEAR)
      DIMENSION X(N),Y(N),XB(N),YB(N),DAR(N)
      LCGICAL V1,V2
      ALEN(XX,YY)=SQRT(XX**2+YY**2)
      AREA=C.0
      YMCM=C.0
      XMOM=C.0
      DC 20 I=2,N
      V1=.FALSE.
      V2=.FALSE.
      X1=X(I-1)
      X2=X(I)
      Y1=Y(I-1)
      Y2=Y(I)
      DTHETA=ATAN2(Y1,X1)-ATAN2(Y2,X2)
      IF(ABS(DTHETA).LT.1.E-05) DTHETA=0.0
      DAR(I)=ALEN(X1,Y1)*ALEN(X2,Y2)*SIN(DTHETA)/2.
      IF(DAR(I).NE.0.0) GO TO 30
      XE(I)=X1
      YE(I)=Y1
      GC TC 20
 30   CCNTINUE
      DELX=2.*X1-X2
      IF(DELX.EQ.0.0) V1=.TRUE.
      IF(V1) GO TC 12
      SL1=(2.*Y1-Y2)/DELX
      B1=Y1-SL1*X1
 12   DELX=2.*X2-X1
      IF(DELX.EQ.0.0) V2=.TRUE.
      IF(V2) GO TC 13
      SL2=(2.*Y2-Y1)/DELX
      B2=Y2-SL2*X2
 13   IF(V1.AND.V2) GO TO 25
      IF(V1) GO TO 14
      IF(V2) GO TC 15
      IF(SL1.NE.SL2) GO TO 90
      DAR(I)=0.0
      XE(I)=0.0
      YE(I)=0.0
      GC TC 20
 90   XE(I)=(B2-B1)/(SL1-SL2)
      GC TC 16
 15   XE(I)=X2
 16   YE(I)=SL1*XB(I)+B1
      GC TC 17
 14   XE(I)=X1
      YE(I)=SL2*XE(I)+B2
 17   CCNTINUE
      AREA=AREA+DAR(I)
      XMOM=DAR(I)*YB(I)+XMOM
      YMOM=DAR(I)*XB(I)+YMOM
 20   CCNTINUE
      IF(AREA.NE.0.0) GC TO 31
      XEAR=C.0
```

```
      YBAR=0.0
      RETURN
   31 CONTINUE
      XBAR=YMOM/AREA
      YBAR=XMOM/AREA
      RETURN
   25 WRITE (6,102)
  102 FORMAT(1H ,'ERROR 2 IN CENTRO-CANNOT DETERMINE
     &CENTROID OF ELEMENT')
      STOP
      END
      SUBROUTINE WATLIN(X,Y,XB,YB,DAR,N)
      COMMON XOFF,YOFF,THETA,ACG,XCG,YCG,XBEL,YBEL,ABEL
      DIMENSION X(N),Y(N),XB(N),YB(N),DAR(N),XX(2),YY(2),XXB(2),
     *YYB(2),DDAR(2)
      DIMENSION XINC(30),YINC(30),INC(30)
      K=0
      DO 10 I=1,N
      CALL TRANS1(X(I),Y(I),XP,YP)
      IF(YP.EQ.0.0) GO TO 2
      IF(I.EQ.1) GO TO 1
      IF(YP*YPOLD.LT.0.0) GO TO 3
    1 XPOLD=XP
      YPOLD=YP
      GO TO 10
    2 K=K+1
      INC(K)=I
      XINC(K)=X(I)
       YINC(K)=Y(I)
       XPOLD=XP
       YPOLD=YP
       GO TO 10
    3  K=K+1
       INC(K)=I
       YINC(K)=0.0
       XINC(K)=XPOLD+(XP-XPOLD)*YPOLD/(YPOLD-YP)
       CALL TRANS2(XINC(K),YINC(K),XINC(K),YINC(K))
       XPOLD=XP
       YPOLD=YP
   10  CONTINUE
       IF(K.LE.1) GO TO 80
       J=1
       ABUF=0.0
       XBUF=0.0
       YBUF=0.0
   40  AREA=0.0
       XMOM=0.0
       YMOM=0.0
        L=J+1
       N11=INC(J)+1
       N2=INC(L)
       DO 20 I=N11,N2
       AREA=AREA+DAR(I)
       XMOM=XMOM+DAR(I)*YB(I)
       YMOM=YMOM+DAR(I)*XB(I)
    20 CONTINUE
       XX(1)=XINC(J)
       XX(2)=X(N11-1)
       YY(1)=YINC(J)
       YY(2)=Y(N11-1)
```

```
      IEND=1
 15   CALL CENTRO(XX.YY.XXB.YYB.DDAR.2.DUM.XDUM.YDUM)
      AREA=AREA+DDAR(2)
      XMOM=XMOM+YYB(2)*DDAR(2)
      YMOM=YMOM+XXB(2)*DDAR(2)
      IF(IEND.EQ.3) GO TO 17
      IF(IEND.EQ.2) GO TO 16
      XX(1)=X(N2)
      YY(2)=YINC(L)
      XX(2)=XINC(L)
      YY(1)=Y(N2)
      IEND=2
      GO TO 15
 16   XX(1)=XINC(L)
      XX(2)=XINC(J)
      YY(1)=YINC(L)
      YY(2)=YINC(J)
      IEND=3
      GO TO 15
 17   CONTINUE
      XBEL=YMOM/AREA
      YBEL=XMOM/AREA
      CALL TRANS1(XBEL.YBEL.XBE.YBE)
      IF(YBE.LT.0.0) GO TO 30
      ABUF=ABUF+AREA
      XBUF=XBUF+XMOM
      YBUF=YBUF+YMOM
 30   J=J+1
      IF(J.LT.K.AND.K.GT.2) GO TO 40
      AREA=0.0
      XMOM=0.0
      YMOM=0.0
      N1=INC(K)
      N11=N1+1
      IF(INC(K).EQ.N) GO TO 100
      DO 50 I=N11.N
      AREA=AREA+DAR(I)
      XMOM=XMOM+DAR(I)*YB(I)
 50   YMOM=YMOM+DAR(I)*XB(I)
 100  N1=INC(1)
      DO 60 I=2.N1
      AREA=AREA+DAR(I)
      XMOM=XMOM+DAR(I)*YB(I)
 60   YMOM=YMOM+DAR(I)*XB(I)
      XX(1)=XINC(K)
      XX(2)=X(N11-1)
      YY(1)=YINC(K)
      YY(2)=Y(N11-1)
      IEND=1
 21   CALL CENTRO(XX.YY.XXB.YYB.DDAR.2.DUM.XDUM.YDUM)
      AREA=AREA+DDAR(2)
      XMOM=XMOM+DDAR(2)*YYB(2)
      YMOM=YMOM+DDAR(2)*XXB(2)
      IF(IEND.EQ.3) GO TO 23
      IF(IEND.EQ.2) GO TO 22
      XX(1)=X(N1)
      XX(2)=XINC(1)
      YY(1)=Y(N1)
      YY(2)=YINC(1)
      IEND=2
      GO TO 21
```

```
22    XX(1)=XINC(1)
      XX(2)=XINC(K)
      YY(1)=YINC(1)
      YY(2)=YINC(K)
      IEND=3
      GO TO 21
23    CONTINUE
      XBEL=YMOM/AREA
      YBEL=XMOM/AREA
      CALL TRANS1(XBEL,YBEL,XBE,YBE)
      IF(YBE.LT.0.0) GO TO 70
      ABUF=ABUF+AREA
      XBUF=XBUF+XMOM
      YBUF=YBUF+YMOM
70    ABEL=ACG-ABUF
      XBEL=(XCG*ACG-YBUF)/ABEL
      YBEL=(YCG*ACG-XBUF)/ABEL
      GO TO 90
80    ABEL=ACG
      XBEL=XCG
      YBEL=YCG
90    RETURN
      END
      SUBROUTINE TRANS1(X,Y,XP,YP)
      COMMON XOFF,YOFF,THETA,ACG,XCG,YCG,XBEL,YBEL,ABEL
      DATA PI2/1.570796/
      BETA=0.0
      IF(Y.EQ.0.0.AND.X.EQ.0.0) GO TO 1
      BETA=ATAN2(Y,X)
1     GAM=PI2-BETA-THETA
      A=SQRT(X*X+Y*Y)
      XP=A*SIN(GAM)
      YP=A*COS(GAM)+YOFF
      RETURN
      END
      SUBROUTINE TRANS2(XP,YP,X,Y)
      COMMON XOFF,YOFF,THETA,ACG,XCG,YCG,XBEL,YBEL,ABEL
      DATA PI2/1.570796/
      BETA=PI2-THETA-ATAN2(XP,YP-YOFF)
      A=SQRT(XP*XP+(YP-YOFF)**2)
      X=A*COS(BETA)
      Y=A*SIN(BETA)
      RETURN
      END
      SUBROUTINE NEWICE(X,Y,XB,YB,N,XCG,YCG)
      DIMENSION X(N),Y(N),XB(N),YB(N)
      DO 10 I=1,N
      X(I)=X(I)-XCG
10    Y(I)=Y(I)-YCG
      DO 30 I=2,N
      XB(I)=XB(I)-XCG
30    YB(I)=YB(I)-YCG
      XCG=0.0
      YCG=0.0
      RETURN
      END
```

```
C       THIS IS THE MAIN PROGRAM FOR THE DYNAMIC ANALYSIS
C       THE TWO DATA STATEMENTS TAKE THE X CO-ORD AND Y CO-ORD
C       VALUES.  THE YOFF AND THETA IN THE READ STATEMENTS TAKE
C       THE INITIAL Y AND T VALUES. THE DELT IN THE READ STATEMENT
C       IS THE TIME STEP WHICH IS USUALLY TAKEN TO BE 0.2 SEC.  THE
C       TMAX IS THE MAXIMUM TIME FOR THE DYNAMIC ANALYSIS WHICH IS
C       USUALLY TAKEN TO BE 300 SECONDS. THE SCALE IN THE READ
C       STATEMENT IS THE AMOUNT BY WHICH THE ICEBERG WILL BE REDUCED
C       FOR PLOTTING PURPOSES.
        DIMENSION X(N),Y(N),XB(N),YB(N),DAR(N)
        COMMON XOFF,YOFF,THETA,ACG,XCG,YCG,XBEL,YBEL,ABEL
        DATA X/      /
        DATA Y/      /
         DATA N/N/
        DATA AREA,XBAR,YBAR/3*0./
         XOFF=0.0
C        YOFF=0.0
        READ 25,YOFF
        READ 52,TMAX
        READ 53,DELT
        READ 54,THETA
        READ 53,SCALE
   25   FORMAT(F11.6)
   52   FORMAT(F6.0)
   53   FORMAT(F4.1)
   54   FORMAT(F10.7)
        CALL CENTRO(X,Y,XB,YB,DAR,N,ACG,XCG,YCG)
        CALL NEWICE(X,Y,XB,YB,N,XCG,YCG)
        CALL CENTRO(X,Y,XB,YB,DAR,N,ACG,XCG,YCG)
        VEL=0.0
        ANGVEL=0.0
        TIME=0.0
        AIZZ=0.0
        DO 11 I=2,N
   11   AIZZ=AIZZ+(XB(I)**2+YB(I)**2)*DAR(I)
        AIZZ=AIZZ*913.
        WEIGHT=ACG*913.
        K=1
   30   CALL WATLIN(X,Y,XB,YB,DAR,N)
        BUOY=ABEL*1000.
        IF(K.EQ.1) BUOYOL=BUOY
        IF(K.EQ.1) DISP=1.0
        SPRIN=ABS(BUOY-BUOYOL)/ABS(DISP)
        CD=0.0931*2.*SQRT(SPRIN*WEIGHT/9.81)
        F=-CD*VEL
        IF(K.GT.1.AND.BUOY.EQ.BUOYOL) F=3.6E04*VEL*VEL
        IF(K.GT.1.AND.BUOY.EQ.BUOYOL.AND.
      &VEL.GT.0.0) F=-F
        ACCEL=(WEIGHT-BUOY+F)*9.81/WEIGHT
        CALL TRANS1(XCG,YCG,XC,YC)
        CALL TRANS1(XBEL,YBEL,XBE,YBE)
        ARM=XBE-XC
         IF(K.EQ.1) ARMOL=ARM
        IF(K.EQ.1) ROT=1.0
        SPRII=ABS(BUOY*ARM-BUOYOL*ARMOL)/ABS(ROT)
        CA=0.0931*2.*SQRT(SPRII*AIZZ/9.81)
        F=-CA*ANGVEL
        IF(K.GT.1.AND.ARMOL.EQ.ARM) F=1.5E10*ANGVEL*ANGVEL
        IF(K.GT.1.AND.ARMOL.EQ.ARM.AND.
      &ANGVEL.GT.0.0) F=-F
```

```
      ANGACC=(BUOY*ARM+F)/AIZZ*9.81
      DISP=VEL*DELT+ACCEL*DELT*DELT/2.0
      VEL=VEL+ACCEL*DELT
      ROT=ANGVEL*DELT+ANGACC*DELT*)ELT/2.0
      ANGVEL=ANGVEL+ANGACC*DELT
      YOFF=YOFF-DISP
      THETA=THETA+ROT
      TIME=DELT+TIME
  100 FORMAT(1H ,7G13.6)
      K=K+1
      BUOYOL=BUOY
      ARMOL=ARM
       WRITE (6,100) TIME,THETA,ANGVEL,ANGACC,YOFF,VEL,ACCEL
      IF(TIME.LT.TMAX) GO TO 30
      CALL OUTLIN(X,Y,N,SCALE)
      STOP
      END
C     THIS IS THE MAIN PROGRAM FOR THE MELTING ANALYSIS
C     THE TWO DATA STATEMENTS TAKE THE X CO-ORD AND Y CO-ORD
C     VALUES.  THE YOFF AND THETA IN THE READ STATEMENTS TAKE
C     THE INITIAL Y AND T VALUES. THE DELT IN THE READ STATEMENT
C     IS THE TIME STEP WHICH IS USUALLY TAKEN TO BE 0.2 SEC.  THE
C     TMAX IS THE MAXIMUM TIME FOR THE DYNAMIC ANALYSIS WHICH IS
C     USUALLY TAKEN TO BE 300 SECONDS.  THE SCALE IN THE READ
C     STATEMENT IS THE AMOUNT BY WHICH THE ICEBERG WILL BE REDUCED
C     FOR PLOTTING PURPOSES.
      DIMENSION X(N),Y(N),XB(N),YB(N),DAR(N)
      COMMON XOFF,YOFF,THETA,ACG,XCG,YCG,XBEL,YBEL,ABEL
      DATA X/     /
      DATA Y/     /
       DATA N/N/
      DATA AREA,XBAR,YBAR/3*0./
       XOFF=0.0
C      YOFF=0.0
      READ 25,YOFF
      READ 52,TMAX
      READ 53,DELT
      READ 54,THETA
      READ 53,SCALE
   25 FORMAT(F11.6)
   52 FORMAT(F6.0)
   53 FORMAT(F4.1)
   54 FORMAT(F10.7)
      CALL CENTRO(X,Y,XB,YB,DAR,N,ACG,XCG,YCG)
      CALL NEWICE(X,Y,XB,YB,N,XCG,YCG)
      CALL CENTRO(X,Y,XB,YB,DAR,N,ACG,XCG,YCG)
      VEL=0.0
      ANGVEL=0.0
      TIME=0.0
      AIZZ=0.0
      DO 11 I=2,N
   11 AIZZ=AIZZ+(XB(I)**2+YB(I)**2)*DAR(I)
      AIZZ=AIZZ*913.
      WEIGHT=ACG*913.
      K=1
      TIM=0.0
      CALL WATLIN(X,Y,XB,YB,DAR,N)
      N1=0
      N2=0
   31 TIM=TIM+0.5
```

```
      CALL MELTIN(X,Y,N,N1,N2,1800.,2000.,5.0,TIM)
      CALL CENTRO(X,Y,XB,YB,DAR,N,ACG,XCG,YCG)
      CALL NEWICE(X,Y,XB,YB,N,XCG,YCG)
      CALL CENTRO(X,Y,XB,YB,DAR,N,ACG,XCG,YCG)
      IF(TIM.EQ.0.0) GO TO 1000
      AIZZ=0.0
      DO 29 I=2,N
   29 AIZZ=AIZZ+(XB(I)**2+YB(I)**2)*DAR(I)
      AIZZ=AIZZ*913.0
      WEIGHT=ACG*913.0
 30   CALL WATLIN(X,Y,XB,YB,DAR,N)
      BUOY=ABEL*1000.
      IF(K.EQ.1) BUOYOL=BUOY
      IF(K.EQ.1) DISP=1.0
      SPRIN=ABS(BUOY-BUOYOL)/ABS(DISP)
      CD=0.0931*2.*SQRT(SPRIN*WEIGHT/9.81)
      F=-CD*VEL
      IF(K.GT.1.AND.BUOY.EQ.BUOYOL) F=3.6E04*VEL*VEL
      IF(K.GT.1.AND.BUOY.EQ.BUOYOL.AND.
     &VEL.GT.0.0) F=-F
      ACCEL=(WEIGHT-BUOY+F)*9.81/WEIGHT
      CALL TRANS1(XCG,YCG,XC,YC)
      CALL TRANS1(XBEL,YBEL,XBE,YBE)
      ARM=XBE-XC
       IF(K.EQ.1) ARMOL=ARM
      IF(K.EQ.1) ROT=1.0
      SPRII=ABS(BUOY*ARM-BUOYOL*ARMOL)/ABS(ROT)
      CA=0.0931*2.*SQRT(SPRII*AIZZ/9.81)
      F=-CA*ANGVEL
      IF(K.GT.1.AND.ARMOL.EQ.ARM) F=1.5E10*ANGVEL*ANGVEL
      IF(K.GT.1.AND.ARMOL.EQ.ARM.AND.
     &ANGVEL.GT.0.0) F=-F
      ANGACC=(BUOY*ARM+F)/AIZZ*9.81
      DISP=VEL*DELT+ACCEL*DELT*DELT/2.0
      VEL=VEL+ACCEL*DELT
      ROT=ANGVEL*DELT+ANGACC*DELT*DELT/2.0
      ANGVEL=ANGVEL+ANGACC*DELT
      YOFF=YOFF-DISP
      THETA=THETA+ROT
      TIME=DELT+TIME
 100  FORMAT(1H ,7G13.6)
      K=K+1
      BUOYOL=BUOY
      ARMOL=ARM
      IF(TIME.LT.TMAX) GO TO 30
       WRITE (6,100) TIME,THETA,ANGVEL,ANGACC,YOFF,VEL,ACCEL
      TIME=0.0
      IF(TIM.GE.38.0) GO TO 1000
      IF(TIM.LE.3000.0) GO TO 31
 1000 WRITE(6,105) (X(I),I=1,N)
 105  FORMAT(1H ,'X=',G13.6)
      WRITE(6,106) (Y(I),I=1,N)
  106 FORMAT(1H ,'Y=',G13.6)
      CALL OUTLIN(X,Y,N,SCALE)
      STOP
      END
      SUBROUTINE OUT_IN(X,Y,N,SCALE)
C     THE OUTLIN SUBROUTINE PLOTS THE ORIENTATION OF THE
C     ICEBERG DURING EVERY TIME STEP OF MELTING.
      COMMON XOFF,YOFF,THETA,ACG,XCG,YCG,XBEL,YBEL,ABEL
```

```
      DIMENSION X(N),Y(N)
      CALL PLOTS(53,0,-9)
      CALL PLOT(5.,5.,-3)
      CALL PLOT(-5.,0.,3)
      CALL PLOT (5.,0.,2)
      CALL TRANS1(XCG,YCG,XC,YC)
      XB=0.0
      YB=YC/SCALE
      CALL PLOT(XB,YB+.1,3)
      CALL PLOT(XB,YB-.1,2)
      CALL PLOT(XB-.1,YB,3)
      CALL PLOT(XB+.1,YB,2)
      CALL TRANS1(XBEL,YBEL,XBE,YBE)
      XB=(XBE-XC)/SCALE
      YB=YBE/SCALE
      CALL PLOT(XB,YB+.1,3)
      CALL PLOT(XB,YB-.1,2)
      CALL PLOT(XB-.1,YB,2)
      CALL PLOT(XB+.1,YB,2)
      CALL PLOT(XB,YB+.1,2)
   15 IP=3
      DO 20 I=1,N
      CALL TRANS1(X(I),Y(I),XI,YI)
      XI=(XI-XC)/SCALE
      YI=YI/SCALE
      CALL PLOT(XI,YI,IP)
   20 IP=2
      CALL PLOT(0.,0.,999)
      RETURN
      END
C     THE MELTIN SUBROUTINE FINDS THE SHAPE OF THE ICEBERG
C     AFTER EVERY 1800 SECONDS OF MELTING.  FOR ICEBERGS
C     WITH A HOLE SUCH AS THE ONE SHOWN IN FIGURE 20,
C     THE POINTS 2 AND 7 SHOULD BE INPUT TO THE ROUTINE
C     THROUGH N1 AND N2 PARAMETERS.  FOR ICEBERGS WITHOUT A
C     HOLE, EITHER N1 OR N2 SHOULD BE EQUAL TO ZERO.
      SUBROUTINE MELTIN(X,Y,N,N1,N2,DELT,H1,T,TIM)
      COMMON XOFF,YOFF,THETA,ACG,XCG,YCG,XBEL,YBEL,ABEL
      DIMENSION X(N),Y(N)
      LOGICAL CROSS
      DO 10 I=1,N
      CALL TRANS1(X(I),Y(I),XP,YP)
      IF(YP.LE.0.0) GO TO 2
      GO TO 10
    2 IF(I.EQ.(N1-1).OR.I.EQ.N2) GO TO 10
      DELR=H1*T*DELT*(3.29E-09)
      IF(I.EQ.N) GO TO 5
      THET=ATAN2((Y(I)-Y(I+1)),(X(I)-X(I+1)))
      DS=SQRT((X(I+1)-X(I))**2+(Y(I+1)-Y(I))**2)
      RNEW=SQRT(DELR**2+DS**2)
      BETA=ATAN2(DELR,DS)+THET
      Y(I)=Y(I+1)+RNEW*SIN(BETA)
      X(I)=X(I+1)+RNEW*COS(BETA)
   10 CONTINUE
      GO TO 11
    5 THET=ATAN2((Y(N)-Y(2)),(X(N)-X(2)))
      DS=SQRT((X(2)-X(N))**2+(Y(2)-Y(N))**2)
      RNEW=SQRT(DELR**2+DS**2)
      BETA=ATAN2(DELR,DS)+THET
      Y(N)=Y(2)+RNEW*SIN(BETA)
```

```
      X(N)=X(2)+RNEW*COS(BETA)
      X(1)=X(N)
      Y(1)=Y(N)
 11   X(N)=X(1)
      Y(N)=Y(1)
      IF(N1.EQ.0.OR.N2.EQ.0) GO TO 32
      X(N1-1)=X(N2+1)
      Y(N1-1)=Y(N2+1)
      X(N2)=X(N1)
      Y(N2)=Y(N1)
      IF(SQRT((X(N2)-X(N2+1))**2+(Y(N2)-Y(N2+1))**2).GT.
     &0.1) GO TO 32
      WRITE(6,100) TIM
100   FORMAT(1H ,'DISTANCE PROBLEM AT TIME=',G13.6)
      TIM=0.0
      RETURN
 32   CROSS=.FALSE.
      DO 20 I=1,N
      CALL TRANS1(X(I),Y(I),XP,YP)
      IF(YP.LE.0.0) GO TO 3
      GO TO 20
  3   COND=0.0
      IND=0
      IF(N1.EQ.0.OR.N2.EQ.0) GO TO 59
      IF(I.EQ.(N1-1).OR.I.EQ.(N2+1)) CALL
     &CHECK(X,Y,N,IND,COND,N1,N2)
 59   IF(IND.EQ.1) CROSS=.TRUE.
      IF(COND.EQ.1.0) GO TO 20
      IF(I.EQ.1) GO TO 8
      ALPH=ATAN2((Y(I-1)-Y(I)),(X(I-1)-X(I)))
      GO TO 9
  8   ALPH=ATAN2((Y(N-1)-Y(I)),(X(N-1)-X(I)))
  9   CALL GTRANS(X,Y,N,I,ALPH,XX,YY)
      IF(XX.LT.0.0.OR.YY.LT.0.0) GO TO 1000
      IF(I.EQ.N.OR.I.EQ.1) GO TO 25
      IF(I.EQ.N1.OR.I.EQ.N2) GO TO 1000
      X(I)=(X(I+1)+X(I-1))/2.0
      Y(I)=(Y(I+1)+Y(I-1))/2.0
      CROSS=.TRUE.
      WRITE(6,110) TIM,I
      GO TO 1000
 25   X(I)=(X(2)+Y(N-1))/2.0
      Y(I)=(Y(2)+Y(N-1))/2.0
       CROSS=.TRUE.
      WRITE(6,110) TIM,I
110   FORMAT(1H ,'CROSS-OVER OCCURS AT TIME=',G13.6,'I=',I4)
1000  CONTINUE
 20   CONTINUE
      X(N)=X(1)
      Y(N)=Y(1)
      IF(CROSS) GO TO 32
      X(N)=X(1)
      Y(N)=Y(1)
      RETURN
      END
      SUBROUTINE CHECK(X,Y,N,IND,COND,N1,N2)
      DIMENSION X(N),Y(N)
      K=N1-1
      ALPH=ATAN2((Y(K-1)-Y(K)),(X(K-1)-X(K)))
      XD=X(N2+2)-X(K)
```

```
      READ 2000,SCALE
      WEIGHT=ACG*913.0
      KTHE=1
  120 CALL WATLIN(X,Y,XB,YB,DAR,N)
      KY=1
      BUOYOL=ABEL*1000.0
      YOL=YOFF
      YOFF=YOFF-1.0
  100 CALL WATLIN(X,Y,XB,YB,DAR,N)
      KY=KY+1
      BUOY=ABEL*1000.0
      IF((BUOYOL-WEIGHT)*(BUOY-WEIGHT).LT.0.0) NEARY=.TRUE.
      IF(NEARY) GO TO 400
      YOL=YOFF
      BUOYOL=BUOY
      YOFF=YOFF+DYOFF
      GO TO 100
  400 CONTINUE
      YNEW=YOL+(YOFF-YOL)*(WEIGHT-BUOYOL)/(BUOY-BUOYOL)
      IF(ABS(1.0-BUOY/WEIGHT).LT.0.0001) GO TO 1015
      YOL=YOFF
      BUOYOL=BUOY
      YOFF=YNEW
      GO TO 100
 1015 CONTINUE
      CALL TRANS1(XCG,YCG,XC,YC)
      CALL TRANS1(XBEL,YBEL,XBE,YBE)
      ARM=XBE-XC
      WRITE(6,300) KTHE,THETA,ARM
  300 FORMAT(1H ,I5,2G13.6)
      IF(ABS(1.0-THETA/THEOLD).LT.0.0001.OR.ABS(ARM).LT.0.0001
     &) GO TO 170
      IF(KTHE.EQ.1) GO TO 140
      IF(ARMOLD*ARM.LE.0.0.OR.NEAR) GO TO 110
  140 ARMOLD=ARM
      THEOLD=THETA
      THETA=THETA+DTHETA
      IF(THETA.GT.6.2832+DTHETA) GO TO 301
  150 KTHE=KTHE+1
      NEARY=.FALSE.
      YOFF=0.0
      GO TO 120
  110 THENEW=THEOLD-(THETA-THEOLD)*ARMOLD/(ARM-ARMOLD)
      ARMOLD=ARM
      THEOLD=THETA
      THETA=THENEW
      IF(.NOT.NEAR) WRITE(6,160)
  160 FORMAT(1H ,'NEAR A ROOT.BEGIN SEARCH FOR A ROOT')
      NEAR=.TRUE.
      GO TO 150
  170 DADT=(ARM-ARMOLD)/(THETA-THEOLD)
      WRITE(6,180) DADT
  180 FORMAT(1H ,'A ROOT HAS BEEN FOUND WITH DADT=',G13.6)
      CALL OUTLIN(X,Y,N,SCALE)
      NEAR=.FALSE.
      KTHE=0
      GO TO 140
  301 STOP
      END
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VITA AUCTORIS

1959	Born in Tirunelveli, Tamil Nadu, India on May 18.
1975	Received Secondary School Leaving Certificate from The M.D.T. Hindu College School, Tirunelveli, India in May
1976	Completed Pre-University Course at St.Xavier's College Palayamkottai, India in March
1981	Received the degree of Bachelor of Engineering in Mechanical Engineering from Coimbatore Institute of Technology, Coimbatore, University of Madras.
1984	Currently a candidate for the degree of Master of Applied Science in Mechanical Engineering at the University of Windsor, Windsor, Ontario, Canada